Solution for Homework 2

1. (a)

$$(\lambda x.xx)(\lambda y.yx)z \to (\lambda u.uu)(\lambda y.yx)z$$
$$\to (\lambda y.yx)(\lambda y.yx)z$$
$$\to (\lambda u.ux)(\lambda y.yx)z$$
$$\to (\lambda y.yx)xz$$
$$\to xxz$$

(b)

$$(\lambda x.(\lambda y.xy)y)z \to (\lambda x.(\lambda u.xu)y)z$$
$$\to (\lambda x.xy)z$$
$$\to zy$$

(c)

$$((\lambda x.xx)(\lambda y.y))(\lambda y.y) \to ((\lambda y.y)(\lambda y.y))(\lambda y.y)$$
$$\to (\lambda y.y)(\lambda y.y)$$
$$\to \lambda y.y$$

(d)

$$((\lambda x.\lambda y.xy)(\lambda y.y))w \to ((\lambda x.\lambda u.xu)(\lambda y.y))w$$
$$\to (\lambda u.(\lambda y.y)u)w$$
$$\to (\lambda y.y)w$$
$$\to w$$

2. (a) Infinite reduction:

$$(\lambda x.y)((\lambda y.yyy)(\lambda x.xxx)) \to (\lambda x.y)((\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx))$$
$$\to (\lambda x.y)((\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx))$$
$$\to \cdots$$
$$\to (\lambda x.y)(\underbrace{(\lambda x.xxx)\cdots(\lambda x.xxx)}_{n \text{ times}})$$

Finite reduction:

$$(\lambda x.y)((\lambda y.yyy)(\lambda x.xxx)) \to y$$

(b) As seen above, the finite reduction reduces to y.

3.

$$SKK = (\lambda x.\lambda y.\lambda z.(xz(yz)))(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)$$
$$\to (\lambda x.\lambda u.\lambda z.(xz(uz)))(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)$$
$$\to (\lambda u.\lambda z.((\lambda x.\lambda y.x)z(uz)))(\lambda x.\lambda y.x)$$
$$\to (\lambda u.\lambda z.((\lambda y.z)(uz)))(\lambda x.\lambda y.x)$$
$$\to (\lambda u.\lambda z.z)(\lambda x.\lambda y.x)$$
$$\to \lambda z.z = I$$

4.

true $= \lambda x.\lambda y.x$

false $= \lambda x.\lambda y.y$

and $= \lambda x.\lambda y.((xy)\textbf{false})$

and false false $= (\lambda u.\lambda v.((uv)\textbf{false}))(\lambda x.\lambda y.y)(\lambda w.\lambda z.z)$
$\rightarrow ((\lambda x.\lambda y.y)(\lambda w.\lambda z.z))\textbf{false}$
$\rightarrow (\lambda y.y)\textbf{false} \rightarrow \textbf{false}$

and false true $= (\lambda u.\lambda v.((uv)\textbf{false})(\lambda x.\lambda y.y)(\lambda w.\lambda z.w)$
$\rightarrow ((\lambda x.\lambda y.y)(\lambda w.\lambda z.w))\textbf{false}$
$\rightarrow (\lambda y.y)\textbf{false} \rightarrow \textbf{false}$

and true false $= (\lambda u.\lambda v.((uv)\textbf{false})(\lambda w.\lambda z.w)(\lambda x.\lambda y.y)$
$\rightarrow ((\lambda w.\lambda z.w)(\lambda x.\lambda y.y))\textbf{false}$
$\rightarrow \lambda x.\lambda y.y \rightarrow \textbf{false}$

and true true $= (\lambda u.\lambda v.((uv)\textbf{false})(\lambda w.\lambda z.w)(\lambda x.\lambda y.x)$
$\rightarrow ((\lambda w.\lambda z.w)(\lambda x.\lambda y.x))\textbf{false}$
$\rightarrow (\lambda z.(\lambda x.\lambda y.x))\textbf{false}$
$\rightarrow (\lambda x.\lambda y.x) \rightarrow \textbf{true}$

5. The definition says that F and $\lambda x.Fx$ are *extensionally equivalent* if x does not occur free in F.

(a)

$F = \text{square}$

$\lambda x.Fx = \lambda x.(\text{square } x)$

square is a function of x, so x is bound in F. Thus, by definition, these expressions are extensionally equivalent.

(b)

$F = \lambda y.(fxy)$

$\lambda x.Fx = \lambda x.((\lambda y.(fxy))x)$

x is free in F. Thus, by definition, these expressions are not extensionally equivalent.