Solutions to the Midterm Exam

Solution 1:
This is one of many possible solutions. The first triangle should be drawn in CCW order.

```gl
glBegin( GL_TRIANGLE_STRIP );
glVertex2f( a.x, a.y );
glVertex2f( d.x, d.y );
glVertex2f( c.x, c.y ); // draws adc
eglVertex2f( e.x, e.y ); // draws dce
glVertex2f( b.x, b.y ); // draws ceb
glEnd( );
```

Solution 2: We will assume that we are already in MODELVIEW mode. Conceptually we want to draw the shape, then translate it to (2,3). Next, we want to draw the shape, then rotate it 180 degrees about the origin (or equivalently scale it by -1 along both x and y) and finally translate it to (5,5). Following OpenGL’s manner of doing things, we reverse the sequence of operations in the code. To keep the first transformation from affecting the second, we enclose each in its own push-pop pair.

```gl
glPushMatrix( ); // save the state again
glTranslatef( 2, 3, 0 ); // translate to (2,3)
drawL( ); // draw shape translated
glPopMatrix( ); // restore stack contents

eglPushMatrix( ); // save stack contents
glTranslatef( 5, 5, 0 ); // translate to (5,5)
glRotatef( 180, 0, 0, 1 ); // rotate 180 degrees about z
drawL( ); // draw shape rotated and translated
glPopMatrix( ); // restore stack contents
```

Solution 3: A positive pitch is a rotation about the x-axis and tilts the plane up. A positive roll is a rotation about the y-axis and tilts the plane so that the pilot’s right wing tip is a bit lower and his left wing tip is a bit higher. A positive yaw is a rotation about the z-axis and causes the plane to point to the left.

Solution 4:

Key-frame animation: Main animator draws a few key frames and others artists or computers draw interpolating frames. Provides high artistic control of the results but is quite labor intensive.

Procedural animation: Specify the motion (e.g. through joint angles) as some function of time. Best for natural/physical objects (balls bouncing, trees blowing in the breeze), especially if combined with randomization. Can take some work to produce realistic models.

Motion capture: Record motion by attaching virtual reality trackers on a person. Good method for acquiring specific human motions.

Solution 5:

(a) The center of mass is the midpoint, whose coordinates are \((x_1 + x_2)/2, (y_1 + y_2)/2\).
(b) The center point in this case is the origin \((0,0)\). So, applying the definition we have:

\[
I = \rho \int_{y=-h/2}^{h/2} \int_{x=-w/2}^{w/2} (x^2 + y^2) \, dx \, dy = \rho \int_{y=-h/2}^{h/2} \left[ \frac{x^3}{3} + xy^2 \right]_{x=-w/2}^{w/2} \, dy
\]

\[
= \rho \int_{y=-h/2}^{h/2} 2 \left( \frac{w^3}{24} + \frac{w}{2} y^2 \right) \, dy = \rho \int_{y=-h/2}^{h/2} \left( \frac{w^3}{12} + wy^2 \right) \, dy
\]

\[
= \rho \left[ \frac{w^3y}{12} + \frac{wy^3}{3} \right]_{y=-h/2}^{h/2} = 2\rho \left( \frac{w^3h}{24} + \frac{wh^3}{24} \right) = \frac{mwh}{12} (w^2 + h^2)
\]

as desired.

(c) The center of mass is the weighted centers of the two masses, where the weight factors are proportional to the masses of the two rectangles. The \(1 \times 3\) rectangle has its center of mass at \((3/2, 5/2)\). The \(2 \times 2\) rectangle has its center of mass at \((3, 2)\). The ratio of the two masses is \((1 \cdot 3) : (2 \cdot 2) = 3 : 4\). Thus the final center of mass is

\[
\frac{3}{7} \left( \frac{3}{2}, \frac{5}{2} \right) + \frac{4}{7} (3,2) = \left( \frac{9}{14}, \frac{15}{14} \right) + \left( \frac{12}{7}, \frac{8}{7} \right) = \left( \frac{33}{14}, \frac{31}{14} \right).
\]

**Solution 6:** Best-first search is the simplest. It expands the node that is closest to the goal. It tends to expand the smallest number of nodes if the path to the goal is nearly obstacle free, but it is not guaranteed to find the shortest path. Dijkstra’s algorithm is guaranteed to compute the shortest path, but does so by looking in all directions (without consideration of where the goal is) and so tends to expand a large number of nodes. A*-search also always computes shortest paths, but because it favors expanding nodes that are close to the goal, it tends to expand fewer nodes than Dijkstra’s algorithm in practice.

**Solution 7:**

(a) Here is one possible solution. Consider a coordinate system where the origin is placed at a point along the left river bank, and the \(y\)-axis points in the direction of the river’s flow. The orientation of the log is described as a CCW angle \(\theta\), where the angle \(\theta = 0\) corresponds to the log being aligned horizontally (with the \(x\)-axis).

(b) Given a configuration point \((x,y,\theta)\) its endpoints are \((x + (L/2)\cos \theta, y + (L/2)\sin \theta)\) and \((x - (L/2)\cos \theta, y - (L/2)\sin \theta)\).

(c) A point \((x,y)\) in the river lies between the left and right banks, if and only if \(0 < x < W\). (Note that the \(y\)-coordinate has no effect.) From part (b), it follows that a log lies in the river if and only if

\[
0 < x + (L/2)\cos \theta < W \quad \text{and} \quad 0 < x - (L/2)\cos \theta < W.
\]

Put another way, if we define the following C-obstacle functions:

\[
f^+(x,y,\theta) = \max(x + (L/2)\cos \theta, x - (L/2)\cos \theta)
\]

\[
f^-(x,y,\theta) = \min(x + (L/2)\cos \theta, x - (L/2)\cos \theta).
\]

Then a configuration point \((x,y,\theta)\) corresponds to a log on the river if and only if \(f^+(x,y,\theta) < W\) and \(f^-(x,y,\theta) > 0\).