Lecture 16:
Rational Numbers

Last time:
1. Aliasing and Mutability
2. Floating Point calculations

Today:
1. Example class development: Rational Numbers
Definition of a Rational Number

- What is a rational number?
- As a decimal it either terminates or repeats a pattern:
  - 1.75
  - 0.242935353535
- As a fraction, it can be represented as a fraction of two integers.
Today we will start an extended example

- We will implement a class, Rational, for (immutable) rational numbers
- The class will include
  - Constructors
  - Arithmetic operations (+, -, *, /)
  - toString
  - Comparisons (equals, compareTo)
“Lowest Terms”?

- How do we represent the fraction 20/60?
  - Reduce to lowest terms.
- Given a fraction p/q, how do you put it into lowest terms?
- Method
  - Find **greatest common divisor (gcd)** of p, q
    - gcd of p, q: largest number that divides both p, q
    - Euclid’s algorithm (beyond scope of this lecture) performs this if p, q are both positive
  - Replace p/q by (p/gcd) / (q/gcd)
- Example
  - Consider 18/24
  - gcd of 18 and 24 is 6
  - So 18/24 = (18/6) / (24/6) = 3/4
Hints

● Come up with representative test cases
● Intertwine implementation and testing
  ● Do constructors and getters first, then test
  ● Implement “related operations”, then test
● Rerun each test (even ones for previously tested methods) when you test
  ● This is called regression testing
  ● Useful for detecting changes that may invalidate previous test results!
  ● Easy to set up in Eclipse
● Use debugger to track down sources of errors in tests
Rational Numbers (continued): Arithmetic Operations

- What you remember from middle / high school
  \[ \frac{p}{q} + \frac{s}{t} = \frac{p \cdot t + q \cdot s}{q \cdot t} \]
  \[ \frac{p}{q} \cdot \frac{s}{t} = \frac{p \cdot s}{q \cdot t} \]
  \[ \frac{p}{q} - \frac{s}{t} = \frac{p}{q} + \left( -\frac{s}{t} \right) \]
  \[ \frac{1}{\frac{p}{q}} = \frac{q}{p} \]
  \[ \frac{\frac{p}{q}}{\frac{s}{t}} = \frac{p}{q} \cdot \frac{t}{s} = \frac{p}{q} \cdot \left( \frac{1}{\frac{s}{t}} \right) \]

We will focus on these two cases.
Comparisons

- \( \frac{p}{q} = \frac{s}{t} \) if
  - \( \frac{p}{q}, \frac{s}{t} \) are in lowest terms, and
  - \( p = q \) and \( s = t \)

- \( \frac{p}{q} < \frac{s}{t} \) if \( p \cdot t < q \cdot s \)

We will focus on this case.