CMSC 132: Object-Oriented Programming II

Sorting

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Overview

- Comparison sort
  - Bubble sort
  - Selection sort } $O(n^2)$
  - Tree sort
  - Heap sort
  - Quick sort
  - Merge sort } O(n log(n))
- Linear sort
  - Counting sort
  - Bucket (bin) sort
  - Radix sort } O(n)

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Sorting

- Goal
  - Arrange elements in predetermined order
    - Based on key for each element
  - Derived from ability to compare two keys by size
- Properties
  - Stable ⇒ relative order of equal keys unchanged
    - Stable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
    - Unstable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
  - In-place ⇒ uses only constant additional space
  - External ⇒ can efficiently sort large # of keys

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Sorting

- Comparison sort
  - Only uses pairwise key comparisons
  - Proven lower bound of O(n log(n))
- Linear sort
  - Uses additional properties of keys

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Bubble Sort

- Approach
  1. Iteratively sweep through shrinking portions of list
  2. Swap element x with its right neighbor if x is larger
- Performance
  - $O(n^2)$ average / worst case

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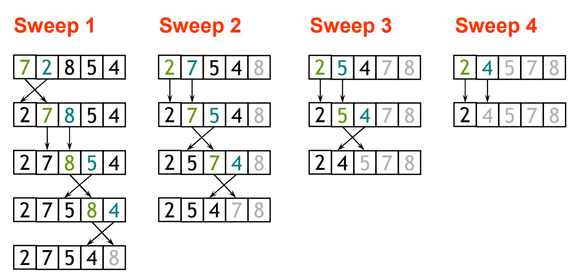
Bubble Sort Example



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Bubble Sort Code

```
void bubbleSort(int[ ] a) {
   int outer, inner;
   for (outer = a.length - 1; outer > 0; outer--)
      for (inner = 0; inner < outer; inner++)
         if (a[inner] > a[inner + 1])
            swap(a, inner, inner+1);
}

void swap(int a[ ], int x, int y) {
   int temp = a[x];
   a[x] = a[y];
   a[y] = temp;
}
```

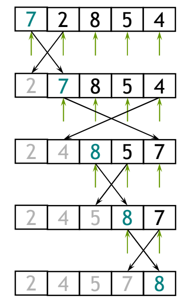
Sweep through array

Swap with right neighbor if larger

Swap array elements at positions x & y

Selection Sort

- Approach
  1. Iteratively sweep through shrinking portions of list
  2. Select smallest element found in each sweep
  3. Swap smallest element with front of current list
- Performance
  - O(n^2) average / worst case
- Example



Selection Sort Code

```
void selectionSort(int[] a) {
      int outer, inner, min;
      for (outer = 0; outer < a.length - 1; outer++) {
         min = outer;
         for (inner = outer + 1; inner < a.length; inner++) {
            if (a[inner] < a[min]) {
               min = inner;
            }
         }
         swap(a, outer, min);
      }
}
```

Sweep through array

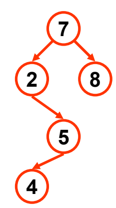
Find smallest element

Swap with smallest element found

Tree Sort

- Approach
  1. Insert elements in binary search tree
  2. List elements using inorder traversal
- Performance
  - Binary search tree
    - O(n log(n)) average case
    - O(n^2) worst case
  - Balanced binary search tree
    - O(n log(n)) average / worst case
- Example

Binary search tree

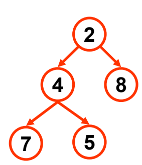


{ 7, 2, 8, 5, 4 }

Heap Sort

- Approach
  1. Insert elements in heap
  2. Remove smallest element in heap, repeat
  3. List elements in order of removal from heap
- Performance
  - O(n log(n)) average / worst case
- Example

Heap



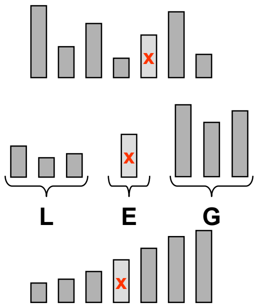
{ 7, 2, 8, 5, 4 }

Quick Sort

- Approach
  1. Select pivot value (near median of list)
  2. Partition elements (into 2 lists) using pivot value
  3. Recursively sort both resulting lists
  4. Concatenate resulting lists
  - For efficiency pivot needs to partition list evenly
- Performance
  - O(n log(n)) average case
  - O(n^2) worst case

Quick Sort Algorithm

1. If list below size K
   - Sort w/ other algorithm
2. Else pick pivot x and partition S into
   - L elements < x
   - E elements = x
   - G elements > x
3. Quicksort L & G
4. Concatenate L, E & G
   - If not sorting in place



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Quick Sort Code

```
void quickSort(int[] a, int x, int y) {
    int pivotIndex;
    if ((y - x) > 0) {
      pivotIndex = partionList(a, x, y);
      quickSort(a, x, pivotIndex - 1);
      quickSort(a, pivotIndex+1, y);
    }
}
```

Lower end of array region to be sorted

Upper end of array region to be sorted

```
int partionList(int[] a, int x, int y) {
     … // partitions list and returns index of pivot
}
```

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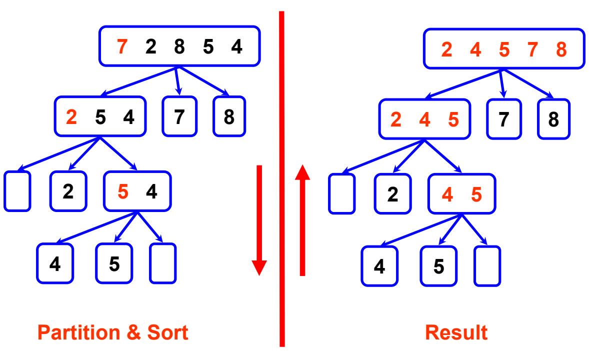
Quick Sort Example



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Quick Sort Code

```
int partitionList(int[] a, int x, int y) {
    int pivot = a[x];
    int left = x;
    int right = y;
    while (left < right) {
      while ((a[left] < pivot) && (left < right))
         left++;
      while (a[right] > pivot)
         right--;
       if (left < right)
         swap(a, left, right);
    }
    swap(a, x, right);
    return right;
}
```

Use first element as pivot

Partition elements in array relative to value of pivot

Place pivot in middle of partitioned array

return index of pivot

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Merge Sort

- Approach
  1. Partition list of elements into 2 lists
  2. Recursively sort both lists
  3. Given 2 sorted lists, merge into 1 sorted list
     a) Examine head of both lists
     b) Move smaller to end of new list
- Performance
  - O(n log(n)) average / worst case

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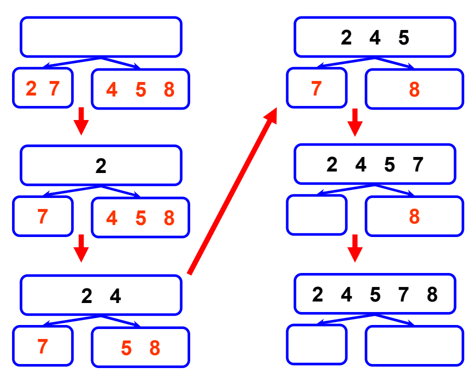
Merge Example



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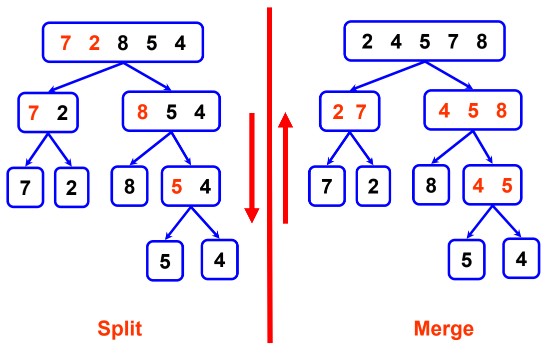
Merge Sort Example



Merge Sort Code

```
void mergeSort(int[] a, int x, int y) {
    int mid = (x + y) / 2;
    if (y == x) return;
    mergeSort(a, x, mid);
    mergeSort(a, mid+1, y);
    merge(a, x, y, mid);
}
void merge(int[] a, int x, int y, int mid) {
    … // merges 2 adjacent sorted lists in array
}
```

Lower end of array region to be sorted

Upper end of array region to be sorted

Merge Sort Code

```
void merge (int[] a, int x, int y, int mid) {
  int size = y - x;
  int left = x;
  int right = mid+1;
  int[] tmp; int j;
  for (j = 0; j < size; j++) {
    if (left > mid) tmp[j] = a[right++];
    else if (right > y) || (a[left] < a[right])
      tmp[j] = a[left++];
    else tmp[j] = a[right++];
    }
  for (j = 0; j < size; j++)
    a[x+j] = tmp[j];
}
```

Upper end of 1st array region

Lower end of 1st array region

Upper end of 2nd array region

Copy smaller of two elements at head of 2 array regions to tmp buffer, then move on

Copy merged array back

Counting Sort

- Approach
  1. Sorts keys with values over range 0..k
  2. Count number of occurrences of each key
  3. Calculate # of keys ≤ each key
  4. Place keys in sorted location using # keys counted
     - If there are x keys ≤ key y
     - Put y in x^{th} position
     - Decrement x in case more instances of key y
- Properties
  - O(n + k) average / worst case

Counting Sort Example

- Original list

7	2	8	5	4
0	1	2	3	4

- Count

0	0	1	0	1	1	0	1	1
0	1	2	3	4	5	6	7	8

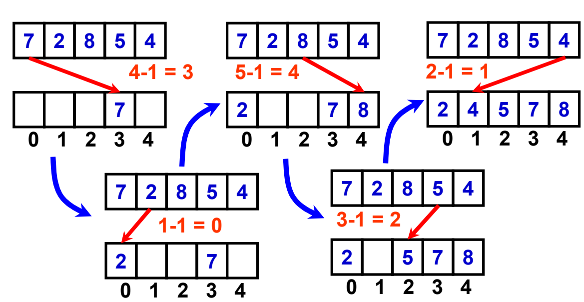
- Calculate # keys ≤ value

0	0	1	1	2	3	3	4	5
0	1	2	3	4	5	6	7	8

Counting Sort Example

- Assign locations

0	0	1	1	2	3	3	4	5
0	1	2	3	4	5	6	7	8



Counting Sort Code

```
void countSort(int[] a, int k) {  // keys have value 0…k
   int[] b; int[] c; int i;
   for (i = 0; i ≤ k; i++)          // initialize counts
      c[i] = 0;
   for (i = 0; i < a.size(); i++)  // count # keys
      c[a[i]]++;
   for (i = 1; i ≤ k; i++)          // calculate # keys ≤ value i
      c[i] = c[i] + c[i-1]
   for (i = a.size()-1; i > 0; i--) {
      b[c[a[i]]-1] = a[i];          // move key to location
      c[a[i]]--;                    // decrement # keys ≤ a[i]
   }
   for (i = 0; i < a.size(); i++)  // copy sorted list back to a
      a[i] = b[i];
}
```

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Bucket (Bin) Sort

- Approach
  1. Divide key interval into k equal-sized subintervals
  2. Place elements from each subinterval into bucket
  3. Sort buckets (using other sorting algorithm)
  4. Concatenate buckets in order
- Properties
  - Pick large k so can sort n / k elements in O(1) time
  - O(n) average case
  - O(n^2) worst case
    - If most elements placed in same bucket and sorting buckets with O(n^2) algorithm

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Bucket Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231
2. Bucket based on 1st digit, then sort bucket
   - 192, 144, 152 ⇒ 144, 152, 192
   - 253, 231 ⇒ 231, 253
   - 552 ⇒ 552
   - 623 ⇒ 623
   - 752 ⇒ 752
3. Concatenate buckets
   - [144, 152, 192] [231, 253] [552] [623] [752]

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Radix Sort

- Approach
  1. Decompose key C into components C_1, C_2, … C_d
     - Component d is least significant
     - Each component has values over range 0..k
  2. For each key component i = d down to 1
     - Apply linear sort based on component C_i (sort must be stable)
  - Example key components
     - Letters (string), digits (number)
- Properties
  - O(d × (n+k)) ≈ O(n) average / worst case

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Radix Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231
2. Sort on 3rd digit (counting sort from 0-9)
   - 231, 192, 152, 752, 552, 623, 253, 144
3. Sort on 2nd digit (counting sort from 0-9)
   - 623, 231, 144, 152, 752, 552, 253, 192
4. Sort on 1st digit (counting sort from 0-9)
   - 144, 152, 192, 231, 253, 552, 623, 752

Compare with: counting sort from 144-752

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Sorting Properties

Name	Comparison Sort	Avg Case Complexity	Worst Case Complexity	In Place	Can be Stable
Bubble	√	O(n^2)	O(n^2)	√	√
Selection	√	O(n^2)	O(n^2)	√	√
Tree	√	O(n log(n))	O(n^2)		
Heap	√	O(n log(n))	O(n log(n))		
Quick	√	O(n log(n))	O(n^2)	√	
Merge	√	O(n log(n))	O(n log(n))		√
Counting		O(n)	O(n)		√
Bucket		O(n)	O(n^2)		√
Radix		O(n)	O(n)		√

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Sorting Summary

- **Many different sorting algorithms**
- **Complexity and behavior varies**
- **Size and characteristics of data affect algorithm**