CMSC 132:
Object-Oriented Programming II

Sorting

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Overview

Comparison sort
- Bubble sort
- Selection sort
- Tree sort
- Heap sort
- Quick sort
- Merge sort

\[ O(n^2) \]

Linear sort
- Counting sort
- Bucket (bin) sort
- Radix sort

\[ O(n) \]
Sorting

Goal

- Arrange elements in **predetermined order**
- Based on key for each element
- Derived from ability to **compare two keys by size**

Properties

- **Stable** ⇒ relative order of **equal** keys unchanged
  - **Stable:** \( 3, 1, 4, 3, 3, 2 \rightarrow 1, 2, 3, 3, 3, 4 \)
  - **Unstable:** \( 3, 1, 4, 3, 3, 2 \rightarrow 1, 2, 3, 3, 3, 4 \)
- **In-place** ⇒ uses only constant additional space
- **External** ⇒ can efficiently sort large # of keys
Sorting

- **Comparison sort**
  - Only uses pairwise key comparisons
  - Proven lower bound of $O(n \log(n))$

- **Linear sort**
  - Uses additional properties of keys
Bubble Sort

Approach
1. Iteratively sweep through shrinking portions of list
2. Swap element $x$ with its right neighbor if $x$ is larger

Performance
- $O(n^2)$ average / worst case
Bubble Sort Example

Sweep 1

7 2 8 5 4
2 7 8 5 4
2 7 8 5 4
2 7 5 8 4
2 7 5 4 8

Sweep 2

2 7 5 4 8
2 7 5 4 8
2 5 7 4 8
2 5 4 7 8
2 4 5 7 8

Sweep 3

2 5 4 7 8
2 5 4 7 8
2 4 5 7 8

Sweep 4

2 4 5 7 8

2 4 5 7 8
void bubbleSort(int[] a) {
    int outer, inner;
    for (outer = a.length - 1; outer > 0; outer--)
        for (inner = 0; inner < outer; inner++)
            if (a[inner] > a[inner + 1])
                swap(a, inner, inner+1);
}

void swap(int a[], int x, int y) {
    int temp = a[x];
    a[x] = a[y];
    a[y] = temp;
}
Selection Sort

Approach
1. Iteratively sweep through shrinking portions of list
2. Select smallest element found in each sweep
3. Swap smallest element with front of current list

Performance
- $O(n^2)$ average / worst case

Example
```c
void selectionSort(int[] a) {
    int outer, inner, min;
    for (outer = 0; outer < a.length - 1; outer++) {
        min = outer;
        for (inner = outer + 1; inner < a.length; inner++) {
            if (a[inner] < a[min]) {
                min = inner;
            }
        }
        swap(a, outer, min);
    }
}
```
Tree Sort

Approach
1. Insert elements in binary search tree
2. List elements using inorder traversal

Performance
- Binary search tree
  - $O(n \log(n))$ average case
  - $O(n^2)$ worst case
- Balanced binary search tree
  - $O(n \log(n))$ average / worst case

Example
Binary search tree
{7, 2, 8, 5, 4}
**Heap Sort**

**Approach**
1. Insert elements in heap
2. Remove smallest element in heap, repeat
3. List elements in order of removal from heap

**Performance**
- O(n log(n)) average / worst case

**Example**

Heap

{7, 2, 8, 5, 4}
Quick Sort

Approach
1. Select pivot value (near median of list)
2. Partition elements (into 2 lists) using pivot value
3. Recursively sort both resulting lists
4. Concatenate resulting lists
   - For efficiency pivot needs to partition list evenly

Performance
- \( O(n \log(n)) \) average case
- \( O(n^2) \) worst case
Quick Sort Algorithm

1. If list below size K
   - Sort w/ other algorithm

2. Else pick pivot $x$ and partition S into
   - $L$ elements $< x$
   - $E$ elements $= x$
   - $G$ elements $> x$

3. Quicksort $L$ & $G$

4. Concatenate $L$, $E$ & $G$
   - If not sorting in place
Quick Sort Code

```c
void quickSort(int[] a, int x, int y) {
    int pivotIndex;
    if ((y - x) > 0) {
        pivotIndex = partionList(a, x, y);
        quickSort(a, x, pivotIndex - 1);
        quickSort(a, pivotIndex+1, y);
    }
}
```

```c
int partionList(int[] a, int x, int y) {
    ... // partitions list and returns index of pivot
}
```
Quick Sort Example

Partition & Sort

Result

7 2 8 5 4

2 5 4 7 8

2 5 4

4 5

2 4 5

4 5

2 4 5

7 8
int partitionList(int[] a, int x, int y) {
    int pivot = a[x];
    int left = x;
    int right = y;
    while (left < right) {
        while ((a[left] < pivot) && (left < right))
            left++;
        while (a[right] > pivot)
            right--;
        if (left < right)
            swap(a, left, right);
    }
    swap(a, x, right);
    return right;
}
Merge Sort

Approach
1. Partition list of elements into 2 lists
2. Recursively sort both lists
3. Given 2 sorted lists, merge into 1 sorted list
   a) Examine head of both lists
   b) Move smaller to end of new list

Performance
- \( O(n \log(n)) \) average / worst case
Merge Example
Merge Sort Example

Split

Merge
void mergeSort(int[] a, int x, int y) {
    int mid = (x + y) / 2;
    if (y == x) return;
    mergeSort(a, x, mid);
    mergeSort(a, mid+1, y);
    merge(a, x, y, mid);
}

void merge(int[] a, int x, int y, int mid) {
    ... // merges 2 adjacent sorted lists in array
}
void merge (int[] a, int x, int y, int mid) {
    int size = y - x;
    int left = x;
    int right = mid + 1;
    int[] tmp; int j;
    for (j = 0; j < size; j++) {
        if (left > mid) tmp[j] = a[right++];
        else if (right > y) || (a[left] < a[right])
            tmp[j] = a[left++];
        else tmp[j] = a[right++];
    }
    for (j = 0; j < size; j++)
        a[x+j] = tmp[j];
}
Counting Sort

**Approach**
1. Sorts keys with values over range 0..k
2. Count number of occurrences of each key
3. Calculate # of keys \( \leq \) each key
4. Place keys in sorted location using # keys counted
   - If there are \( x \) keys \( \leq \) key \( y \)
   - Put \( y \) in \( x^{th} \) position
   - Decrement \( x \) in case more instances of key \( y \)

**Properties**
- \( O(n + k) \) average / worst case
Counting Sort Example

- Original list

<table>
<thead>
<tr>
<th>7</th>
<th>2</th>
<th>8</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- Count

<table>
<thead>
<tr>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

- Calculate # keys ≤ value

<table>
<thead>
<tr>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>
Counting Sort Example

Assign locations

0 0 1 1 2 3 3 4 5

0 1 2 3 4 5 6 7 8

7 2 8 5 4

4-1 = 3

0 1 2 3 4

7 2 8 5 4

5-1 = 4

0 1 2 3 4

7 2 8 5 4

2-1 = 1

0 1 2 3 4

7 2 8 5 4

1-1 = 0

0 1 2 3 4

2 7 8 5 4

3-1 = 2

0 1 2 3 4

2 5 7 8
Counting Sort Code

```c
void countSort(int[] a, int k) {
    int[] b; int[] c; int i;
    for (i = 0; i ≤ k; i++) // initialize counts
        c[i] = 0;
    for (i = 0; i < a.size(); i++) // count # keys
        c[a[i]]++;
    for (i = 1; i ≤ k; i++) // calculate # keys ≤ value i
        c[i] = c[i] + c[i-1]
    for (i = a.size()-1; i > 0; i--) {
        b[c[a[i]]-1] = a[i]; // move key to location
        c[a[i]]--; // decrement # keys ≤ a[i]
    }
    for (i = 0; i < a.size(); i++) // copy sorted list back to a
        a[i] = b[i];
}
```
Bucket (Bin) Sort

Approach
1. Divide key interval into $k$ equal-sized subintervals
2. Place elements from each subinterval into bucket
3. Sort buckets (using other sorting algorithm)
4. Concatenate buckets in order

Properties
- Pick large $k$ so can sort $n / k$ elements in $O(1)$ time
- $O( n )$ average case
- $O( n^2 )$ worst case
  - If most elements placed in same bucket and sorting buckets with $O( n^2 )$ algorithm
Bucket Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231

2. Bucket based on 1st digit, then sort bucket
   - 192, 144, 152  ⇒  144, 152, 192
   - 253, 231  ⇒  231, 253
   - 552  ⇒  552
   - 623  ⇒  623
   - 752  ⇒  752

3. Concatenate buckets
   - 144, 152, 192  231, 253  552  623  752
Radix Sort

Approach

1. Decompose key $C$ into components $C_1, C_2, \ldots, C_d$
   - Component $d$ is least significant
   - Each component has values over range $0 \ldots k$
2. For each key component $i = d$ down to $1$
   - Apply linear sort based on component $C_i$
     (sort must be stable)

Example key components
- Letters (string), digits (number)

Properties
- $O(d \times (n+k)) \approx O(n)$ average / worst case
Radix Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231

2. Sort on 3rd digit (counting sort from 0-9)
   - 231, 192, 152, 752, 552, 623, 253, 144

3. Sort on 2nd digit (counting sort from 0-9)
   - 623, 231, 144, 152, 752, 552, 253, 192

4. Sort on 1st digit (counting sort from 0-9)
   - 144, 152, 192, 231, 253, 552, 623, 752

Compare with: counting sort from 144-752
# Sorting Properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Comparison Sort</th>
<th>Avg Case Complexity</th>
<th>Worst Case Complexity</th>
<th>In Place</th>
<th>Can be Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble</td>
<td>√</td>
<td>O(n^2)</td>
<td>O(n^2)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Selection</td>
<td>√</td>
<td>O(n^2)</td>
<td>O(n^2)</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Tree</td>
<td>√</td>
<td>O(n log(n))</td>
<td>O(n^2)</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Heap</td>
<td>√</td>
<td>O(n log(n))</td>
<td>O(n log(n))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick</td>
<td>√</td>
<td>O(n log(n))</td>
<td>O(n^2)</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Merge</td>
<td>√</td>
<td>O(n log(n))</td>
<td>O(n log(n))</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Counting</td>
<td></td>
<td>O(n)</td>
<td>O(n)</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Bucket</td>
<td></td>
<td>O(n)</td>
<td>O(n^2)</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>Radix</td>
<td></td>
<td>O(n)</td>
<td>O(n)</td>
<td></td>
<td>√</td>
</tr>
</tbody>
</table>
Sorting Summary

- Many different sorting algorithms
- Complexity and behavior varies
- Size and characteristics of data affect algorithm