Fall 2007

CMSC 451: Homework 3  Clyde Kruskal

Due at the start of class Thursday, October 18, but you may hand it in Tuesday, October 23, 2007.

**Problem 1.** Do Exercise 2 on page 246 of Kleinberg and Tardos.

**Problem 2.** Do Exercise 3 on pages 246-7 of Kleinberg and Tardos.

**Problem 3.** Assume we could multiply two $3 \times 3$ matrices with $k$ multiplications (and some large number of additions).

a. How fast (just the order, i.e. do not worry about the constants) is a new “fancy” matrix multiplication algorithm based on this fact?

b. How small does $k$ have to be for this algorithm to be asymptotically faster than the “standard” ($O(n^3)$) algorithm? Justify.

c. How small does $k$ have to be for this algorithm to be asymptotically faster than Strassen’s ($O(n^\log_2 7)$) algorithm? Justify.

**Problem 4.** We are going to multiply the two polynomials $A(x) = 3 - 5x$ and $B(x) = 2 + 6x$ to produce $C(x) = a + bx + cx^2$ in three different ways. Do this by hand, and show your work.

(a) Multiply $A(x) \times B(x)$ algebraically.

(b) Evaluate $A$ and $B$ at the three (real) roots of unity 1, $i$, $-1$. (Note that we could use any three values.)

(i) Multiply the values at the three roots of unity to form the values of $C(x)$ at the three roots.

(ii) Plug 1, $i$, $-1$ into $C(x) = a + bx + cx^2$ to form three simultaneous equations with three unknowns.

(iv) Solve for $a$, $b$, $c$.

(c) Evaluate $A$ and $B$ at the fourth (real) 4th roots of unity 1, $i$, $-1$, $-i$.

(ii) Multiply the values at the four 4th roots to form the values of $C(x)$ at the four 4th roots.

(iii) Create the polynomial $D(x) = C(1) + C(i)x + C(-1)x^2 + C(-i)x^3$.

(iv) Evaluate $D$ at the four 4th roots of unity 1, $i$, $-1$, $-i$.

(v) Use these values to construct $C(x)$.

**Problem 5.** Use the FFT algorithm to evaluate $f(x) = 6 - 5x + 3x^2 + 2x^3 - 7x^4 - 3x^5 + x^6 + 2x^7$ at the eight 8th roots of unity mod 17. You may stop using recursion when evaluating a linear function $(a + bx)$, which is easier to do directly. The eight 8th roots of unity mod 17 are 1, 2, 4, 8, 16, 15, 13, 9; it is easier to calculate with 1, 2, 4, 8, -1, -2, -4, -8. Do this by hand, and show your work.