

Due at the start of class Thursday, October 18, but you may hand it in Tuesday, October 23, 2007.

Problem 1. Do Exercise 2 on page 246 of Kleinberg and Tardos.

Problem 2. Do Exercise 3 on pages 246-7 of Kleinberg and Tardos.

Problem 3. Assume we could multiply two 3×3 matrices with k multiplications (and some large number of additions).

- a. How fast (just the order, i.e. do not worry about the constants) is a new “fancy” matrix multiplication algorithm based on this fact?
- b. How small does k have to be for this algorithm to be asymptotically faster than the “standard” ($O(n^3)$) algorithm? Justify.
- c. How small does k have to be for this algorithm to be asymptotically faster than Strassen’s ($O(n^{\lg 7})$) algorithm? Justify.

Problem 4. We are going to multiply the two polynomials $A(x) = 3 - 5x$ and $B(x) = 2 + 6x$ to produce $C(x) = a + bx + cx^2$ in three different ways. Do this by hand, and show your work.

- (a) Multiply $A(x) \times B(x)$ algebraically.
- (b)
 - (i) Evaluate A and B at the three (real) roots of unity $1, i, -1$. (Note that we could use any three values.)
 - (ii) Multiply the values at the three roots of unity to form the values of $C(x)$ at the three roots.
 - (iii) Plug $1, i, -1$ into $C(x) = a + bx + cx^2$ to form three simultaneous equations with three unknowns.
 - (iv) Solve for a, b, c .
- (c)
 - (i) Evaluate A and B at the four (real) 4th roots of unity $1, i, -1, -i$.
 - (ii) Multiply the values at the four 4th roots to form the values of $C(x)$ at the four 4th roots.
 - (iii) Create the polynomial $D(x) = C(1) + C(i)x + C(-1)x^2 + C(-i)x^3$.
 - (iv) Evaluate D at the four 4th roots of unity $1, i, -1, -i$.
 - (v) Use these values to construct $C(x)$.

Problem 5. Use the FFT algorithm to evaluate $f(x) = 6 - 5x + 3x^2 + 2x^3 - 7x^4 - 3x^5 + x^6 + 2x^7$ at the eight 8th roots of unity mod 17. You may stop using recursion when evaluating a linear function ($a + bx$), which is easier to do directly. The eight 8th roots of unity mod 17 are 1, 2, 4, 8, 16, 15, 13, 9; it is easier to calculate with 1, 2, 4, 8, -1, -2, -4, -8. Do this by hand, and show your work.