CMSC 451:Fall 2007 Clyde Kruskal

Practice Problems for the Final Exam

Disclaimer: These are practice problems for the upcoming final exam. This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

Problem 1. Suppose you are given three strings of characters: $X = x_1x_2...x_m$, $Y = y_1y_2...y_n$, and $Z = z_1z_2...z_{m+n}$. Z is an interleaving of X and Y if Z can be formed by interleaving the characters of X and Y to form Z in a way that maintains the left-to-right ordering of the characters from X and Y. For example, assume X=on and Y=to, then the following strings are all of the interleavings of X and Y: onto, otno, oton, tono, toon.

Given X, Y, Z, m, and n, we would like to know if Z is an interleaving of X and Y.

(a) Formulate this as a dynamic programming problem. Write a recurrence. Note that since we are asking a YES-NO question it is easiest to do this using Boolean algebra.

(b) Based on Part (a), give an efficient dynamic programming algorithm. Analyze its running time.

Problem 2. For each of the following give the answer "True," "False" or "Not known to science". Let MST denote the decision problem form of the minimum spanning tree: given a weighted graph G, and integer X does G have a spanning tree of total weight at most X?

(a) There is a language L that is in NP, but is neither in P nor is NP-complete.

(b) MST $\leq_P$ 3-SAT.

(c) 3-SAT $\leq_P$ MST.

(d) Suppose some known NP-hard problem can be solved in polynomial time, but this problem is not in NP. Then P = NP.

(e) Suppose some known NP-hard problem *cannot* be solved in polynomial time, but this problem is not in NP. Then P $\neq$ NP.

(f) Suppose that $L_1 \leq_P L_2$, and a factor-2 approximation algorithm is known for L_2. Then there is a constant factor approximation (perhaps with a different factor) for L_1.

(g) The following problem is NP-complete: Given an undirected graph G, does it have a clique of size 99?

Problem 3. Consider the following problem, called HP2: Given an undirected graph G, such that the number of vertices in G is a power of two, does G have a Hamiltonian path?

Show that HP2 is NP-complete, by showing (a) that HP2 is in NP, and (b) HP (Hamiltonian path) is reducible to HP2. Prove the correctness of your reduction. (Hint: Reduction from HP.)

Problem 4. Give an $O(n + e)$ time algorithm to determine whether a DAG $G = (V, E)$ has a Hamiltonian path. (Hint: Use DFS.)

Problem 5. Prove that the following problem, called the *high-degree independent set problem* (HDIS) is NP-complete. Given an undirected graph G with n vertices and an integer k, does G have an independent set of size k, which consists entirely of vertices of degree at least $n/2$? (Hint: Reduction from Independent Set.)

Problem 6. Prove that the following problem, called the *acyclic subgraph problem* (AS) is NP-complete. Given a directed graph $G = (V, E)$ and an integer k, determine whether G contains a subset V' of k vertices such that the induced subgraph on V' is acyclic. The *induced subgraph* on V' is the subgraph $G' = (V', E')$ whose vertex set is V', and for which $(u, v) \in E'$ if $u, v \in V'$ and $(u, v) \in E$. (Hint: Reduction from Independent Set. Think of a reduction that maps undirected edges to directed cycles.)