CMSC 498M: Chapter 8a
Game Physics

Reading:
- Physics for Game Developers, by David M. Bourg, 2002.

Overview:
- Basic physical quantities: Mass, center of mass, moment of inertia.
- Kinematics for particles: Position, velocity, acceleration.
- Kinematics for rigid bodies: Angular velocity, angular acceleration.
- Forces: Springs and friction.

Game Physics: Basic Concepts

Basic Issues:

Kinematics: The study of motion (ignoring forces). How does acceleration affect velocity? How does velocity affect position?
- Particle: A point-mass. Body rotation ignored.
- Rigid body: Rotation of the body needs to be considered.

Force: Objects change motion only when forces are applied.
- Contact vs. field forces: Hitting a baseball vs. gravity or magnetism.
- Torque: Force that induces rotation.
- Environmental sources: Friction, buoyancy, drag/lift.

Kinetics: (also called Dynamics) The effect of force on motion.

Non-rigid Objects:
- Joints and constraints: Rag-doll physics, mass-spring systems.
- Flexible objects: Soft bodies, meshes, cloth, hair.

Complex Motion: Fluid dynamics, smoke, particle systems.

Collisions: Detection and response.
Applications of Physics in Games

Flight simulators
Sports/Racing
Combat Simulation
many others...

Flight simulators: (e.g., for flight simulators)
- Full 3D motion modeling.
- Effects due to lift, drag, turbulence.
- Control issues (how do ailerons, flaps, rudder affect motion).

Sports/Racing: (e.g., for racing games)
- Friction, road resistance, breaking, skidding, drifting.
- Effects of road banking.
- Crashing and tumbling.

Kinetics: How do forces effect the motion of an object.

Projectiles: (e.g., bullets, cannon balls)
- Body rotation may be negligible or totally ignored.
- Effects due to gravity, wind, air resistance.

Aircraft: (e.g., for flight simulators)
- Full 3D motion modeling.
- Effects due to lift, drag, turbulence.
- Control issues (how do ailerons, flaps, rudder affect motion).

Ships: (and floating objects)
- Buoyancy and flotation.
- Motion resistance and fluid dynamics.

Cars: (e.g. for racing games)
- Friction, road resistance, breaking, skidding, drifting.
- Effects of road banking.
- Crashing and tumbling.
Overview

Basic physical quantities

Kinematics for particles

Kinematics for rigid bodies

Force

Game Physics: Mathematics

Units and measures:
- Type-checking for physicists.
- English or metric? Important to keep these straight.

3-dimensional geometry:
- Basic representations: Scalars, points, vectors, matrices, tensors (we won't discuss tensors).
- 3D geometric processing:
  - Linear/affine transformations.
  - Dot and cross product of vectors.
  - Rotation and quaternions (we may discuss this later).

Calculus: Although derivations involve understanding of calculus, most computations just use basic constructs.
- Differential calculus: finite differences.
- Integral calculus: finite summations.
- Differential equations: simulated using small time steps.
Physical Quantities and Units

Basic Quantities:

<table>
<thead>
<tr>
<th>Measure</th>
<th>English</th>
<th>Metric (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Slug</td>
<td>Kilogram (kg)</td>
</tr>
<tr>
<td>Length</td>
<td>Foot (ft)</td>
<td>Meter (m)</td>
</tr>
<tr>
<td>Time</td>
<td>Second (s)</td>
<td>Second (s)</td>
</tr>
</tbody>
</table>

Examples of Other Quantities:

<table>
<thead>
<tr>
<th>Measure</th>
<th>English</th>
<th>Metric (SI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>Pound (lb)</td>
<td>Newton (N)</td>
</tr>
<tr>
<td>Pressure</td>
<td>lb/ft²</td>
<td>N/m²</td>
</tr>
<tr>
<td>Velocity</td>
<td>ft/s</td>
<td>m/s</td>
</tr>
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</table>

Common Notation and Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Dimensions</th>
<th>English</th>
<th>Metric (SI)</th>
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<tr>
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<td>L, s</td>
<td>L</td>
<td>feet (ft)</td>
<td>meters (m)</td>
</tr>
<tr>
<td>Mass</td>
<td>m</td>
<td>M</td>
<td>slug</td>
<td>kilogram (kg)</td>
</tr>
<tr>
<td>Time</td>
<td>T, t</td>
<td>T</td>
<td>seconds (s)</td>
<td>seconds (s)</td>
</tr>
<tr>
<td>Force</td>
<td>F</td>
<td>M(L/T²)</td>
<td>pound (lb)</td>
<td>Newton (N)</td>
</tr>
<tr>
<td>Acceleration, linear</td>
<td>a</td>
<td>L/T²</td>
<td>ft/s²</td>
<td>m/s²</td>
</tr>
<tr>
<td>Acceleration, angular</td>
<td>α</td>
<td>radian/T²</td>
<td>radian/s²</td>
<td>radian/s²</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>I</td>
<td>ML²</td>
<td>lb-ft-s²</td>
<td>kg-m²</td>
</tr>
<tr>
<td>Moment (torque)</td>
<td>M</td>
<td>M(L²/T²)</td>
<td>ft-lb</td>
<td>N-m</td>
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<tr>
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<td>ρ</td>
<td>M/L³</td>
<td>slug/ft³</td>
<td>kg/m³</td>
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<tr>
<td>Pressure</td>
<td>P</td>
<td>M/(LT²)</td>
<td>lb/ft²</td>
<td>N/m²</td>
</tr>
<tr>
<td>Velocity, linear</td>
<td>V, v</td>
<td>L/T</td>
<td>ft/s</td>
<td>m/s</td>
</tr>
<tr>
<td>Velocity, angular</td>
<td>ω</td>
<td>radian/T</td>
<td>radian/s</td>
<td>radian/s</td>
</tr>
</tbody>
</table>
**Isaac Newton’s Laws of Motion: (circa 1687)**

**Law I:** A body tends to remain at rest or continue to move in a straight line at constant velocity, unless it is acted upon by an external force. (Inertia)

**Law II:** The acceleration of a body is proportional to the resultant force acting on the body, and this acceleration is in the same direction as the force. \( F = ma \)

**Law III:** For every force acting on a body (action) there is an equal and oppositely directed reacting force (reaction).

**Relevance:** Most of game physics involves implementing Newton’s laws.

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**Rigid Body Physics**

**Rigid Bodies:**
- No moving parts, no hinges, no flexibility.
- Very simple to analyze/model.
- Unlike a point masses, need to consider rotation.
- Complex objects can often be modeled as systems of rigid bodies.

**Rigid Body Properties:**

**Mass:** (scalar)
- the amount of matter.
- the degree of resistance to change in motion (inertial mass).

**Center of Mass** (or gravity): (point/vector)
- central point about which rotations occur.
- need not lie within the body (if the body is nonconvex).

**Moment of Inertia:** (scalar)
- the resistance to rotational motion about a given axis (scalar form).
Rigid Body Properties: Mass

**Mass**: Total amount of matter.

*Exact*: For a volume $V$ of constant density $\rho$, integrate the density times differential volume elements.

\[
m = \int \rho \, dV = \rho \int dV = (\text{density}) \cdot (\text{volume})
\]

*Approximate*: In practice, this integral is approximated by the sum of masses of small volume elements that make up a more complex object (can allow variable densities as well):

\[
m \approx \sum \rho_i \cdot v_i
\]

Rigid Body Properties: Center of Mass

**Center of Mass**: A point (vector) quantity, equals the mean coordinate values weighted by mass. Let $c = (c_x, c_y, c_z)$, where:

*Exact*: For each axis, integrate the differential mass elements, times its coordinate value.

\[
\begin{align*}
    c_x &= \frac{\int x \, dm}{m} \\
    c_y &= \frac{\int y \, dm}{m} \\
    c_z &= \frac{\int z \, dm}{m}
\end{align*}
\]

*Approximate*: Sum over small mass elements:

\[
\begin{align*}
    c_x &\approx \frac{\sum x m}{m} \\
    c_y &\approx \frac{\sum y m}{m} \\
    c_z &\approx \frac{\sum z m}{m}
\end{align*}
\]
Rigid Body Properties: Moment of Inertia

Moment of Inertia: For a given center point and rotation axis, represents the resistance to rotation about this point/axis. Let's consider rotation about the origin and the z-axis.

Exact: Integrate the squared distance from each mass element to the z-axis.

\[ I_z = \int r_z^2 \, dm = \int (x^2 + y^2) \, dm \]

Approximate: Sum over small mass elements:

\[ I_z \approx \sum r_z^2 \, m_i = \sum (x_i^2 + y_i^2) \, m_i \]

Change of Origin: Can we update the moment of inertia if the rotation axis remains the same but the origin changes?

Parallel Axis Theorem: Consider a body of mass m. Let \( I_c \) be the moment of inertia about the body's center of mass (CoM) \( c \), and let \( I_p \) be the moment of inertia for a parallel rotation axis, but about an arbitrary point \( p \) such that the distance \( |pc| \) is \( d \). Then:

\[ I_p = I_c + md^2 \]

Corollary: Rotation about the center of mass has the lowest moment of inertia (for any fixed axis of rotation).
Rigid Body Properties: Moment of Inertia

**Change of Rotation Axis:** Can we update the moment of inertia if the origin remains the same but the rotation axis changes?

**Simple Answer:** No! The (scalar) moment of inertia does not contain enough information.

**Complex Answer:** There is a more complex structure, called the inertia tensor, that implicitly stores the moment of inertia with respect to all possible axes. (We won’t discuss it.)

Overview

Basic physical quantities

**Kinematics for particles**

Kinematics for rigid bodies

Force


**Kinematics: Speed and Velocity**

**Speed and Velocity:**

**Average Speed:** Let \( s \) denote the object’s position and \( t \) denote time. Assuming motion along a line, speed is the change in position \( \Delta s \) over some time interval \( \Delta t \):

\[
v = \frac{\Delta s}{\Delta t}.
\]

**Units:** Speed is often measured in feet per second (ft/s), miles per hour (mi/h), meters per second (m/s), etc.

**Instantaneous speed:** If speed varies with time, we need to consider the limit for differential (infinitely small) time intervals:

\[
v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.
\]

**Velocity:** Is a vector-valued quantity, whose magnitude is the speed and whose direction indicates the direction of motion.

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**Kinematics: Acceleration**

**Acceleration:** Change in speed over time.

**Average Acceleration:** Change in speed \( \Delta v \) over some time \( \Delta t \).

\[
a = \frac{\Delta v}{\Delta t}.
\]

**Instantaneous acceleration:** If speed varies with time, we need to consider the limit for differential (infinitely small) time intervals:

\[
a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.
\]

**Units:** Acceleration is measured in ft/s\(^2\), mi/h\(^2\), m/s\(^2\), etc.

**Example:** A car goes from 0 to 60 m/h in 4.2 seconds.

The average acceleration in ft/s\(^2\) is:

\[
a = \frac{\Delta v}{\Delta t} = \frac{60 \text{ mi}}{4.2 \text{ s}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} = \frac{60 \times 5280}{4.2 \times 3600} \approx 21 \text{ ft/s}^2.
\]
Kinematics: Relationships

Relating Position, Velocity, and Acceleration:

Position and Velocity: By definition, \( v(t) = \frac{ds}{dt} \). Suppose that an object moves from position \( s_0 \) to \( s_1 \) during the time period \( t_0 \) to \( t_1 \). We have:

\[
\int_{s_0}^{s_1} ds = \int_{t_0}^{t_1} v(t) \, dt \\
\Delta s = s_1 - s_0 = \int_{t_0}^{t_1} v(t) \, dt
\]

We sometimes drop the parameter \( t \) and just write \( \Delta s \) here.

Velocity and Acceleration: By similar argument we have:

\[ \Delta v = v_1 - v_0 = \int_{t_0}^{t_1} a(t) \, dt \]

All three: We also have:

\[
a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad \text{and} \quad v(t) \, dv = a(t) \, ds
\]

Example: Constant Acceleration

Constant Acceleration: What is the position, as a function of time, of an object moving with constant acceleration \( a \)?

Start: At time \( t = 0 \) the object is at position \( s_0 \) with velocity \( v_0 \).

Question: At time \( t \geq 0 \), what is the object's position, \( s(t) \)?

Analysis: We observed earlier that:

\[ \Delta v = v_1 - v_0 = \int_{0}^{t} a \, dt \]

that is \( v(t) - v_0 = \int_{0}^{t} a \, dt \)

Since acceleration is constant this yields\( v(t) = v_0 + at \) Using the fact that \( v(t) \, dt = ds \), we have:

\[
\int_{s_0}^{s(t)} ds = \int_{0}^{t} v(t) \, dt = \int_{0}^{t} (v_0 + at) \, dt \\
s(t) - s_0 = \left[ v_0 t + \frac{at^2}{2} \right]_{0}^{t} = v_0 t + \frac{at^2}{2} \\
s(t) = s_0 + v_0 t + \frac{at^2}{2}
\]

Now I remember why I hate physics.
Overview

Basic physical quantities

Kinematics for particles

**Kinematics for rigid bodies**

Force

Angular Velocity

**Rigid Body Rotation:**
- So far we have only discussed translation, which would be fine if all objects were treated as particles (point-mass).
- For a complete understanding, we must consider rotation.
- Rotation occurs about:
  - the object's center of mass and
  - some axis of rotation (which may change depending on forces).

**Plane Kinematics:**
- All rotation occurs about a fixed axis of rotation in 3-space (i.e., on a plane orthogonal to that axis).
- Good enough for many 2½-dimensional games (e.g., Mario Bros).

**General Kinematics:**
- 3D rotation: Euler angles and quaternions.
Plane Kinematics: Object State

Local Coordinate Frame:
- We select a coordinate frame so that the z-axis is aligned with the axis of rotation.

Object Orientation: To specify an object’s location in space:
- Location of its center of mass: (x, y, z) coordinates in local frame.
- Current angle of rotation \( \Omega \) relative to the z-axis.

Plane Kinematics: Angular Velocity & Acceleration

Each of the principal quantities for translational motion has its counterpart in angular motion.

<table>
<thead>
<tr>
<th>Translational Motion</th>
<th>Angular Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>Position</td>
<td>s</td>
</tr>
<tr>
<td>Velocity</td>
<td>v</td>
</tr>
<tr>
<td>Acceleration</td>
<td>a</td>
</tr>
</tbody>
</table>
### Plane Kinematics: Angular Velocity & Acceleration

#### Definitions and Relations

<table>
<thead>
<tr>
<th>Translational motion</th>
<th>Angular motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = \frac{ds}{dt} )</td>
<td>( \omega = \frac{d\Omega}{dt} )</td>
</tr>
<tr>
<td>( a = \frac{dv}{dt} = \frac{d^2s}{dt^2} )</td>
<td>( \alpha = \frac{d\omega}{dt} = \frac{d^2\Omega}{dt^2} )</td>
</tr>
<tr>
<td>( s = \int v , dt )</td>
<td>( \Omega = \int \omega , dt )</td>
</tr>
<tr>
<td>( v = \int a , dt )</td>
<td>( \omega = \int \alpha , dt )</td>
</tr>
<tr>
<td>( v , dv = a , ds )</td>
<td>( \omega , d\omega = \alpha , d\Omega )</td>
</tr>
</tbody>
</table>

**Constant Acceleration Formulas**

<table>
<thead>
<tr>
<th>Translational motion</th>
<th>Angular motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) = v_0 + a \cdot t )</td>
<td>( \omega(t) = \omega_0 + \alpha \cdot t )</td>
</tr>
<tr>
<td>( s(t) = s_0 + v_0 \cdot t + \frac{a \cdot t^2}{2} )</td>
<td>( \Omega(t) = \Omega_0 + \omega_0 \cdot t + \frac{\alpha \cdot t^2}{2} )</td>
</tr>
</tbody>
</table>

### Overview

- Basic physical quantities
- Kinematics for particles
- Kinematics for rigid bodies
  - Force
Force

Kinematics:
- The study of motion in the absence of force.

Kinetics:
- How do we integrate forces with mass to determine motion?

Force Basics:
- Contact force: from impact, friction, buoyancy, pressure.
- Field force: from gravity and electromagnetism.
- Newton's Third Law: Forces come in pairs (action and reaction). We will usually compute just one, and the other is its negation.

Examples:
- Springs - useful for handling collisions.
- Friction - braking, skidding, sliding.
- Dampers - motion resistors
- Buoyancy - floating

Springs

Springs:
- Elements (usually particles) joined by elastic structures, called springs.
- Springs assumed to follow Hooke's Law (given below).
- Complex mass-spring systems can be used to model complex objects, such as cloth.
**Springs**

**Hooke's Law:** for an ideal (linear) spring.
- Let \( p_1 \) and \( p_2 \) be two particles connected by a spring.
- Let \( L = \| p_1 - p_2 \| \) be the distance between \( p_1 \) and \( p_2 \).
- Let \( u \) = unit length directional vector from \( p_1 \) to \( p_2 \).
- Then the spring force is (vector quantity):
  \[ F_{\text{spring}} = k (L - L_{\text{rest}}) u, \]
  where:
  - \( L_{\text{rest}} \) = the length of the spring at rest, and
  - \( k \) = spring constant. (Units: force/length, e.g. lb/ft or Newton/m.)

**Application:**
- \( F_{\text{spring}} \) is applied to \( p_1 \).
- \(-F_{\text{spring}}\) is applied to \( p_2 \).
- \( L < L_{\text{rest}} \): repels the points.
- \( L > L_{\text{rest}} \): attracts the points.

**Friction**

**Friction:**
- Friction is a contact force.
- It is directed tangential to the plane of contact.
- It resists force (in the static case) or velocity (in the dynamic case).
- It is a complex phenomenon, which is modeled by Coulomb friction.
Coulomb Friction

Coulomb Friction:
- Consider two objects \( o_1 \) and \( o_2 \) in contact.
- Let \( \mathbf{n} \) be the unit length normal vector to the contact plane.
- Suppose that \( o_1 \) moves at tangential velocity \( \mathbf{v}_t \) relative to \( o_2 \).
- What is the force on \( o_1 \) due to friction? (The force on \( o_2 \) will be the negation of this.)

\[ F_{\text{app}} = F_n + F_t \]

Coulomb Friction

Coulomb Friction:
- Suppose that a force \( F_{\text{app}} \) is applied to \( o_1 \). (This includes gravity, but does not include friction.)
- We can decompose the vector \( F_{\text{app}} \) into two components, one parallel to \( \mathbf{n} \) \( (F_n) \) and one orthogonal \( (F_t) \):
  \[ F_n = (F_{\text{app}} \cdot \mathbf{n}) \mathbf{n} \]
  \[ F_t = F_{\text{app}} - F_n \]
- We will compute the force due to friction \( F_{\text{frict}} \), and the final net force will be:
  \[ F_{\text{net}} = F_{\text{app}} + F_{\text{frict}} \]
- We consider two cases, depending on whether \( o_1 \) is moving.
Coulomb Friction: Static Case

**Static Case:** If \( v_t = 0 \):
- Let \( \mu_s \) be the **static coefficient of friction**. (No units.)
- **Friction force** is given by:
  \[
  F_{\text{frict}} = -\frac{F_t}{|F_t|} \min(\mu_s |F_n|, |F_t|).
  \]
  
  **Case 1:** if \(|F_t| \leq \mu_s |F_n|\), then \( F_{\text{frict}} = -F_t \), implying that the net motion is 0. Object \( o_1 \) **does not move**.
  
  **Case 2:** if \(|F_t| > \mu_s |F_n|\), then \( |F_{\text{frict}}| < |F_t| \), implying that there will be a **nonzero net tangential force** of \( F_t + F_{\text{frict}} \), and thus the object will start to move in the direction of \( F_t \).

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Coulomb Friction: Dynamic Case

**Dynamic Case:** If \( v_t \neq 0 \):
- Let \( \mu_d \) be the **dynamic coefficient of friction**. (No units.)
- **Friction force** is given by:
  \[
  F_{\text{frict}} = -\frac{V}{|V|} \mu_d |F_n|.
  \]
  
  - **Note:** This is negative and tends to **decrease tangential velocity**.
  - This is true irrespective of the direction of \( F_t \).

**Final Net Force:**
- \( F_{\text{net}} = F_{\text{app}} + F_{\text{frict}} \).
Summary

Summary:
- **Basic physical concepts**: Mass, center of mass, moment of inertia.
- **Kinematics for particles**: Position, velocity, acceleration.
- **Kinematics for rigid bodies**: Orientation, angular velocity, angular acceleration.
- **Simple forces**: Springs and friction.