CMSC330: Discussion Material (10/1)

October 1, 2008

1) What language does the following grammar generate:

\[ S \rightarrow aSbS \mid bSaS \mid \epsilon \]

**Answer:** The set of all strings with an equal number of a’s and b’s.

2)

a) Give the grammar that generates the language of the regular expression: \[ 0^*1(0 \mid 1)^* \]

**Answer:**

\[
\begin{align*}
S & \rightarrow A1B \\
A & \rightarrow 0A \mid \epsilon \\
B & \rightarrow 0B \mid 1B \mid \epsilon
\end{align*}
\]

b) Give the leftmost and rightmost derivations of the following strings using the grammar from part a):

i) **1001**

**Answer:**

Leftmost: \[ S \rightarrow A1B \rightarrow 1B \rightarrow 10B \rightarrow 100B \rightarrow 1001B \rightarrow 1001 \]

Rightmost: \[ S \rightarrow A1B \rightarrow A10B \rightarrow A100B \rightarrow A1001B \rightarrow A1001 \rightarrow 1001 \]

ii) **00011**

**Answer:**

Leftmost: \[ S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 000A1B \rightarrow 0001B \rightarrow 00011B \rightarrow 00011 \]

Rightmost: \[ S \rightarrow A1B \rightarrow A11B \rightarrow A11 \rightarrow A011 \rightarrow A0011 \rightarrow A00011 \rightarrow 00011 \]

c) Give the parse trees for the strings in part b).

i) **Answer:**

![Parse tree for 1001](image)
ii) **Answer:**

3) Design a context free grammar for the following language: \( \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k \} \)

**Answer:**

\[
\begin{align*}
S & \rightarrow RC \mid AT \\
R & \rightarrow aRb \mid X \\
X & \rightarrow aA \mid bB \\
T & \rightarrow bTc \mid Y \\
Y & \rightarrow bB \mid cC \\
A & \rightarrow aA \mid \epsilon \\
B & \rightarrow bB \mid \epsilon \\
C & \rightarrow cC \mid \epsilon
\end{align*}
\]

**Note:** The important productions rules here are for the symbols R and T. R enforces the constraint \( i \neq j \) and T enforces the constraint \( j \neq k \). The start symbol just ensures that one or the other must be true for the string to be in the language of this grammar.

4) Given the following grammar:

\[
S \rightarrow aS \mid aSb \mid \epsilon
\]

a) Is this grammar ambiguous? Why?

**Answer:** Yes. The string \( aab \) has two parse trees, leftmost derivations, and rightmost derivations.

b) Redesign the grammar to be unambiguous.

**Answer:**

\[
\begin{align*}
S & \rightarrow aSb \mid A \\
A & \rightarrow Aa \mid \epsilon
\end{align*}
\]

**Note:** This is the grammar for the language \( a^m b^n \mid m \geq n \)

5) Construct a NFA for the following grammar:

\[
\begin{align*}
S & \rightarrow aA \mid B \\
A & \rightarrow aaB \\
B & \rightarrow bB \mid a
\end{align*}
\]
Answer:

Note: The grammar must be right or left-linear to perform this conversion. This grammar is right-linear.