CMSC330: Discussion Material (10/1)

October 1, 2008

1) What langage does the following grammar generate:

$$S \quad \rightarrow \quad aSbS \mid bSaS \mid \epsilon$$

Answer: The set of all strings with an equal number of a's and b's.

2)

a) Give the grammar that generates the language of the regular expression: **0*1(0 | 1)***
Answer:

$$\begin{array}{lcl} S & \rightarrow & A1B \\ A & \rightarrow & 0A \mid \epsilon \\ B & \rightarrow & 0B \mid 1B \mid \epsilon \end{array}$$

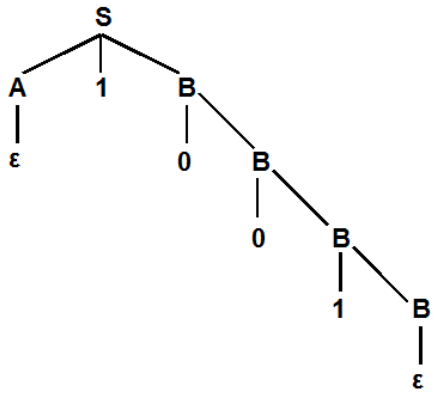
b) Give the leftmost and rightmost derivations of the following strings using the grammar from part a):

i) **1001**
Answer:
Leftmost: $S \rightarrow A1B \rightarrow 1B \rightarrow 10B \rightarrow 100B \rightarrow 1001B \rightarrow 1001$
Rightmost: $S \rightarrow A1B \rightarrow A10B \rightarrow A100B \rightarrow A1001B \rightarrow A1001 \rightarrow 1001$

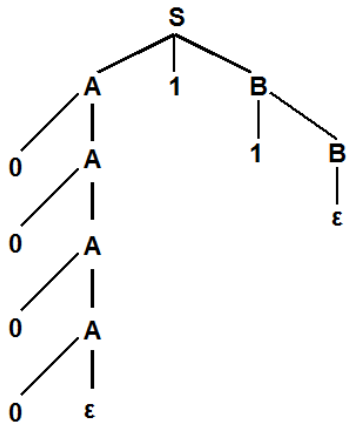
ii) **00011**
Answer:
Leftmost: $S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 000A1B \rightarrow 0001B \rightarrow 00011B \rightarrow 00011$
Rightmost: $S \rightarrow A1B \rightarrow A11B \rightarrow A11 \rightarrow A011 \rightarrow A0011 \rightarrow A00011 \rightarrow 00011$

c) Give the parse trees for the strings in part b).

i) ***Answer:***



ii) ***Answer:***



3) Design a context free grammar for the following language: $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$
Answer:

$$\begin{array}{lcl} S & \rightarrow & RC \mid AT \\ R & \rightarrow & aRb \mid X \\ X & \rightarrow & aA \mid bB \\ T & \rightarrow & bTc \mid Y \\ Y & \rightarrow & bB \mid cC \\ A & \rightarrow & aA \mid \epsilon \\ B & \rightarrow & bB \mid \epsilon \\ C & \rightarrow & cC \mid \epsilon \end{array}$$

Note: The important productions rules here are for the symbols R and T. R enforces the constraint $i \neq j$ and T enforces the constraint $j \neq k$. The start symbol just ensures that one or the other must be true for the string to be in the language of this grammar.

4) Given the following grammar:

$$S \rightarrow aS \mid aSb \mid \epsilon$$

a) Is this grammar ambiguous? Why?
Answer: Yes. The string aab has two parse trees, leftmost derivations, and rightmost derivations.

b) Redesign the grammar to be unambiguous.
Answer:

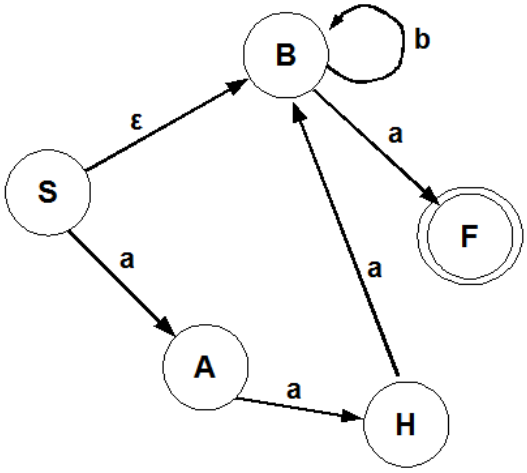
$$\begin{array}{lcl} S & \rightarrow & aSb \mid A \\ A & \rightarrow & Aa \mid \epsilon \end{array}$$

Note: This is the grammar for the language $a^m b^n \mid m \geq n$

5) Construct a NFA for the following grammar:

$$\begin{array}{lcl} S & \rightarrow & aA \mid B \\ A & \rightarrow & aaB \\ B & \rightarrow & bB \mid a \end{array}$$

Answer:



Note: The grammar must be right or left-linear to perform this conversion. This grammar is right-linear.