CMSC330: CFG Practice Problems

1) What language does the following grammar generate?

$$S \quad \rightarrow \quad aaSbb \mid \epsilon$$

2) Given the language $\boldsymbol{L = \{a^i b^j c^{2i+1} d^k \mid i, j, k \geq 0\}}$:

a) Design an unambiguous grammar that generates $\boldsymbol{L}$.

b) Give the leftmost and rightmost derivations of the string **abbcccd** using the grammar from part a).

c) Give the parse trees for the string **abbcccd** based on the grammar of part a).

3) Given the language $\boldsymbol{L = \{a^n b^m a^{n-m} \mid n \geq m \geq 0\}}$

a) Design an unambiguous grammar that generates $\boldsymbol{L}$.

b) Give the leftmost and rightmost derivations of the string **aaabba** using the grammar from part a).

c) Give the parse trees for the string **aaabba** based on the grammar of part a).

4) Given the regular expression $\boldsymbol{R = (a \mid b)^+ b^* a b^*}$:

a) Design a grammar for $\boldsymbol{R}$.

b) Give the leftmost and rightmost derivations of the string **aaabba** using the grammar from part a).

c) Give the parse trees for the string **aaabba** based on the grammar of part a).

5) Draw a NFA/DFA that represents the same language as the following grammar:

$$\begin{array}{lcl} \boldsymbol{S} & \rightarrow & \boldsymbol{01R \mid \epsilon} \\ \boldsymbol{R} & \rightarrow & \boldsymbol{0R \mid T \mid \epsilon} \\ \boldsymbol{T} & \rightarrow & \boldsymbol{0T \mid 0} \end{array}$$

6) Given the language $\boldsymbol{L = \{w \in \{a, b\}^* \mid w}$ has the same number of $\boldsymbol{a}$'s and $\boldsymbol{b}$'s$\boldsymbol{\}}$:

a) Give a~~n unambiguous~~ grammar describing $\boldsymbol{L}$.
Edit: the grammar you write can be ambiguous. It's pretty hard to write an unambiguous one for this problem.
Note: Remember, the $\boldsymbol{a}$'s and $\boldsymbol{b}$'s can be in any order, as long as there are the same number of each.

b) Give a derivation for the string $\boldsymbol{aabb}$ using your grammar from a) and draw the corresponding parse tree.

7) Give ~~an unambiguous~~ grammar describing the following languages:
Edit: the grammars you write can be ambiguous. It's pretty hard to write unambiguous ones for this problem.

a) $\{w \in \{a, b\}^* \mid$ the number of a's in w is **two times** the number of b's$\}$

b) $\{w \in \{a, b\}^* \mid$ the number of a's in w is two times the number of b's **plus one**$\}$

8) Given the regular expression $((1(0 \mid 1)^*) \mid \epsilon)0$:

a) Draw a corresponding DFA.

b) Design a grammar for the language.

c) Describe in words what language is generated by this grammar.

9) Give a grammar for each of the following languages:

a) $\{a^m b^n a^{m+n} \mid m \geq 0$ and $n \geq 1\}$

b) $\{a^m b^n c^p d^q \mid m + n = p + q\}$

c) $\{w \in \{a, b\}^* \mid w$ contains no substrings of the form $ab\}$

d) $\{\gamma_1 \gamma_2 \ldots \gamma_n \gamma_n \ldots \gamma_2 \gamma_1 \mid \gamma_i \in \{a, b\}, 1 \leq i \leq n\}$
Note: This language is describing all strings that are palindromes, that is, they read the same forwards as backwards, with alphabet $\Sigma = \{a, b\}$

10) Design a grammar for the language: $\{a^n b^n a^m b^m \mid m, n \geq 1\} \cup \{a^n b^m a^m b^n \mid m, n \geq 1\}$.

11) The following grammar is an ambiguous grammar for balanced parentheses:

$$S \rightarrow (S) \mid SS \mid \epsilon$$

a) Show that this grammar is ambiguous.

b) Design a new grammar for balanced parentheses that is not ambiguous.