1) What language does the following grammar generate?

\[ S \rightarrow aaSbb | \epsilon \]

2) Given the language \( L = \{a^ib^jca^{2i+1}d^k \mid i, j, k \geq 0\} \):
   a) Design an unambiguous grammar that generates \( L \).
   b) Give the leftmost and rightmost derivations of the string \text{abbcccd} using the grammar from part a).
   c) Give the parse trees for the string \text{abbcccd} based on the grammar of part a).

3) Given the language \( L = \{a^nba^nb^n \mid n \geq m \geq 0\} \):
   a) Design an unambiguous grammar that generates \( L \).
   b) Give the leftmost and rightmost derivations of the string \text{aaabba} using the grammar from part a).
   c) Give the parse trees for the string \text{aaabba} based on the grammar of part a).

4) Given the regular expression \( R = (a | b)^{+}b^{*}ab^{*} \):
   a) Design a grammar for \( R \).
   b) Give the leftmost and rightmost derivations of the string \text{aaabba} using the grammar from part a).
   c) Give the parse trees for the string \text{aaabba} based on the grammar of part a).

5) Draw a NFA/DFA that represents the same language as the following grammar:

\[
\begin{align*}
S & \rightarrow 01R | \epsilon \\
R & \rightarrow 0R | T | \epsilon \\
T & \rightarrow 0T | 0
\end{align*}
\]

6) Given the language \( L = \{w \in \{a, b\}^* \mid w \text{ has the same number of } a \text{'s and } b \text{'s}\} \):
   a) Give an unambiguous grammar describing \( L \).
      Edit: the grammar you write can be ambiguous. It’s pretty hard to write an unambiguous one for this problem.
      \textbf{Note}: Remember, the \( a \)'s and \( b \)'s can be in any order, as long as there are the same number of each.
   b) Give a derivation for the string \text{aabb} using your grammar from a) and draw the corresponding parse tree.
7) Give an unambiguous grammar describing the following languages:
   Edit: the grammars you write can be ambiguous. It’s pretty hard to write unambiguous ones for this problem.
   a) \( \{ w \in \{ a, b \}^* \mid \text{the number of } a\text{'s in } w \text{ is two times the number of } b\text{'s}\} \)
   b) \( \{ w \in \{ a, b \}^* \mid \text{the number of } a\text{'s in } w \text{ is two times the number of } b\text{'s plus one}\} \)

8) Given the regular expression \(((1(0 | 1)^*) | \epsilon)0^*\):
   a) Draw a corresponding DFA.
   b) Design a grammar for the language.
   c) Describe in words what language is generated by this grammar.

9) Give a grammar for each of the following languages:
   a) \( \{ a^m b^n a^{m+n} \mid m \geq 0 \text{ and } n \geq 1 \} \)
   b) \( \{ a^m b^n c^p d^q \mid m + n = p + q \} \)
   c) \( \{ w \in \{ a, b \}^* \mid w \text{ contains no substrings of the form } ab \} \)
   d) \( \{ \gamma_1 \gamma_2 \cdots \gamma_n \gamma_n \cdots \gamma_2 \gamma_1 \mid \gamma_i \in \{ a, b \}, 1 \leq i \leq n \} \)
   Note: This language is describing all strings that are palindromes, that is, they read the same forwards as backwards, with alphabet \( \Sigma = \{ a, b \} \)

10) Design a grammar for the language: \( \{ a^n b^n a^m b^m \mid m, n \geq 1 \} \cup \{ a^n b^m a^m b^n \mid m, n \geq 1 \} \).

11) The following grammar is an ambiguous grammar for balanced parentheses:

\[
S \rightarrow (S) \mid SS \mid \epsilon
\]
   a) Show that this grammar is ambiguous.
   b) Design a new grammar for balanced parentheses that is not ambiguous.