

The following exercises are designed to test your understanding of recursion. The functions are defined using a variant of LISP known as meta-LISP. In order to aid your understanding, the function defined in problem 1 is identical to the one below:

```
drop(x) = if null x then nil
          else (car x) cons drop(cdr x)
```

The idea is that

a x = car x

d x = cdr x

n x = null x

at x = atom x

a.b = a cons b

<a> = a cons nil = a list whose single element is a

a\*b = concatenate lists a and b (i.e. append list b to list a)

reverse[x] = reverses the top level list x. For example reverse[(A B C)] = (C B A). But reverse[((A B C)(D E))] = ((D E)(A B C)).

1. Consider the function drop defined by

```
drop[x] ← if n x then nil else [a x].drop[d x].
```

Compute (by hand) drop[(A B C)]. What does drop do to lists in general?

2. What does the function

```
r2[x] ← if n x then nil else reverse[a x].r2[d x]
```

do to lists of lists? How about

```
r3[x] ← if at x then x else reverse[r4[x]]
```

```
r4[x] ← if n x then nil else r3[a x].r4[d x]?
```

3. Compare the following function with the function r3 of the preceding example:

```
r3'[x] ← if at x then x else r3'[d x]*<r3'[a x]>
```

4. Consider r5 defined by

```
r5[x] ← if n x ∨ n d x then x
```

```
          else [a r5[d x]] . r5[a x . r5[d r5[d x]]].
```

Compute r5[(A B C D)]. What does r5 do in general. Needless to say, this is not a good way of computing this function even though it involves no auxiliary functions.