Graph Implementation

Department of Computer Science
University of Maryland, College Park
Graph Implementation

How do we represent edges?

- **Adjacency matrix**
  - 2D array of neighbors
- **Adjacency list**
  - List of neighbors
- **Adjacency set / map**
  - Set / map of neighbors

Important for very large graphs

- Affects efficiency / storage
Adjacency Matrix

**Representation**

- 2D array
- Position $j, k \Rightarrow$ edge between nodes $n_j, n_k$

**Example**

![Diagram showing an example of an adjacency matrix and a graph representing the same information.]
Adjacency Matrix

Representation (cont.)

- Single array for entire graph
- Undirected graph
  - Only upper / lower triangle matrix needed
  - Since $n_j, n_k$ implies $n_k, n_j$
- Unweighted graph
  - Matrix elements $\Rightarrow$ boolean
- Weighted graph
  - Matrix elements $\Rightarrow$ weight
**Adjacency List/Set**

**Representation**

- For each node, store
  - List/Set of neighbors / successors
    - Linked list
    - Array list
- For weighted graph
  - Also store weight for each edge
  - Using a Map is a good choice
- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes \(a\) & \(b\) need to store each other as neighbor
- For directed graph with edge \((a \rightarrow b)\)
  - Node \(a\) needs to store node \(b\) as neighbor
**Adjacency List**

**Example**

- **Unweighted graph**
  - node 1: \{2, 3\}
  - node 2: \{1, 3, 4\}
  - node 3: \{1, 2, 4, 5\}
  - node 4: \{2, 3, 5\}
  - node 5: \{3, 4, 5\}

- **Weighted graph**
  - node 1: \{2=3.7, 3=5\}
  - node 2: \{1=3.7, 3=1, 4=10.2\}
  - node 3: \{1=5, 2=1, 4=8, 5=3\}
  - node 4: \{2=10.2, 3=8, 5=1.5\}
  - node 5: \{3=3, 4=1.5, 5=6\}
Adjacency Set / Map

Representation

- For each node, store
  - Set or map of neighbors / successors
- For unweighted graph
  - Use set of neighbors
- For weighted graph
  - Use map of neighbors, w/ value = weight of edge
- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes a & b need to store each other as neighbor
- For directed graph with edge \((a \rightarrow b)\)
  - Node a needs to store node b as neighbor
Graph Space Requirements

- **Adjacency matrix**
  - \( \frac{1}{2} N^2 \) entries (for graph with \( N \) nodes, \( E \) edges)
  - Many empty entries for large, sparse graphs

- **Adjacency list**
  - \( 2 \times E \) entries

- **Adjacency set / map**
  - \( 2 \times E \) entries
  - Space overhead per entry
    - Higher than for adjacency list
Graph Time Requirements

- **Adjacency matrix**
  - Can find individual edge \((a,b)\) quickly
  - Examine entry in array \(\text{Edge}[a,b]\)
    - Constant time operation

- **Adjacency list / set / map**
  - Can find all edges for node \((a)\) quickly
  - Iterate through collection of edges for \(a\)
    - On average \(E / N\) edges per node
## Graph Time Requirements

### Average Complexity of operations

For graph with **N** nodes, **E** edges

<table>
<thead>
<tr>
<th>Operation</th>
<th>Adj Matrix</th>
<th>Adj List</th>
<th>Adj Set/Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find edge</td>
<td>O(1)</td>
<td>O(E/N)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Insert edge</td>
<td>O(1)</td>
<td>O(E/N)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Delete edge</td>
<td>O(1)</td>
<td>O(E/N)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Enumerate edges for node</td>
<td>O(N)</td>
<td>O(E/N)</td>
<td>O(E/N)</td>
</tr>
</tbody>
</table>
Choosing Graph Implementations

- **Graph density**
  - Ratio edges to nodes (dense vs. sparse)

- **Graph algorithm**
  - **Neighbor based**
    - For each node X in graph
      - For each neighbor Y of X // adj list faster if sparse
        - doWork( )
  - **Connection based**
    - For each node X in ...
      - For each node Y in ...
        - if (X,Y) is an edge // adj matrix faster if dense
          - doWork( )