

CMSC 132: Object-Oriented Programming II



Graph Implementation

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Graph Implementation

- **How do we represent edges?**
 - **Adjacency matrix**
 - **2D array of neighbors**
 - **Adjacency list**
 - **List of neighbors**
 - **Adjacency set / map**
 - **Set / map of neighbors**

- **Important for very large graphs**
 - **Affects efficiency / storage**

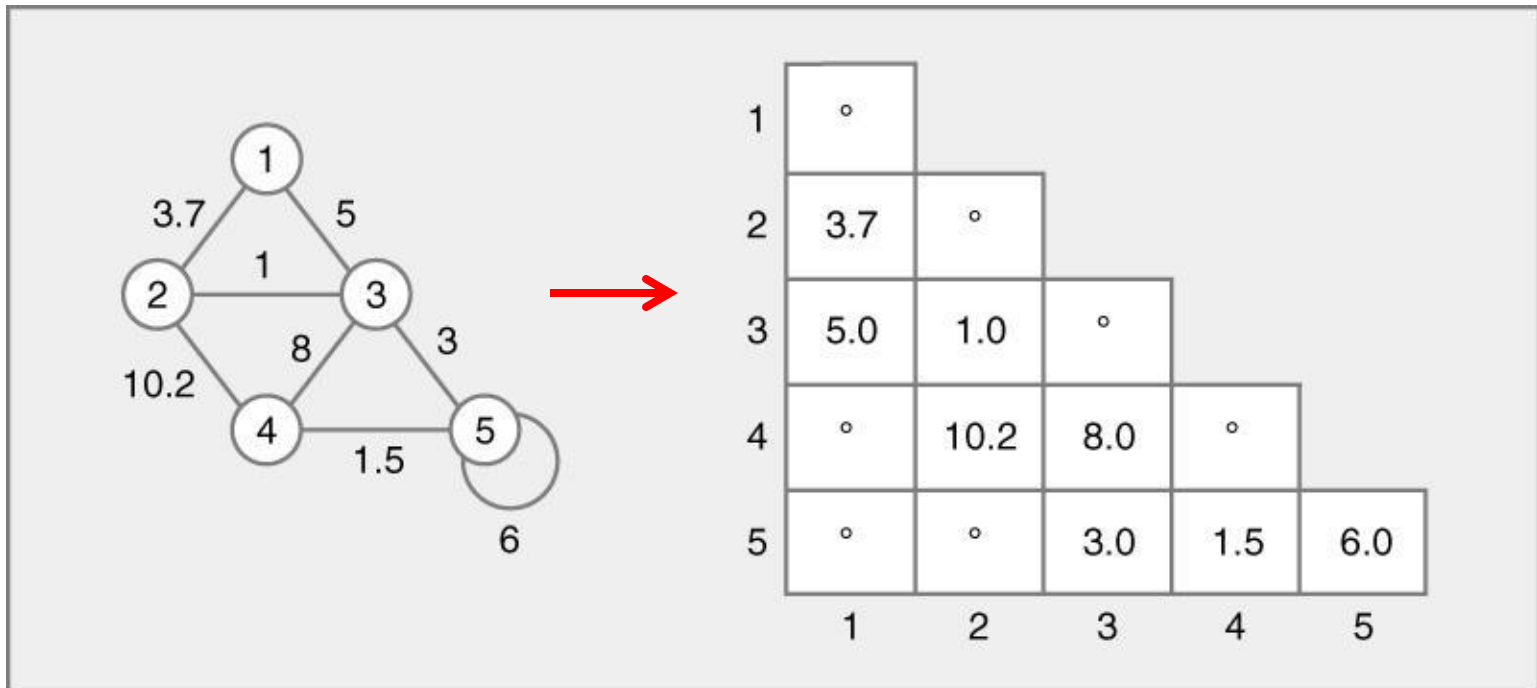
Adjacency Matrix

■ Representation

- 2D array

- Position $j, k \Rightarrow$ edge between nodes n_j, n_k

■ Example



Adjacency Matrix

- **Representation (cont.)**
 - **Single array for entire graph**
 - **Undirected graph**
 - Only upper / lower triangle matrix needed
 - Since n_j, n_k implies n_k, n_j
 - **Unweighted graph**
 - Matrix elements \Rightarrow boolean
 - **Weighted graph**
 - Matrix elements \Rightarrow weight

Adjacency List/Set

■ Representation

■ For each node, store

■ List/Set of neighbors / successors

- Linked list

- Array list

■ For weighted graph

- Also store weight for each edge

- Using a Map is a good choice

■ For undirected graph with edge ($a \leftrightarrow b$)

- Nodes a & b need to store each other as neighbor

■ For directed graph with edge ($a \rightarrow b$)

- Node a needs to store node b as neighbor

Adjacency List

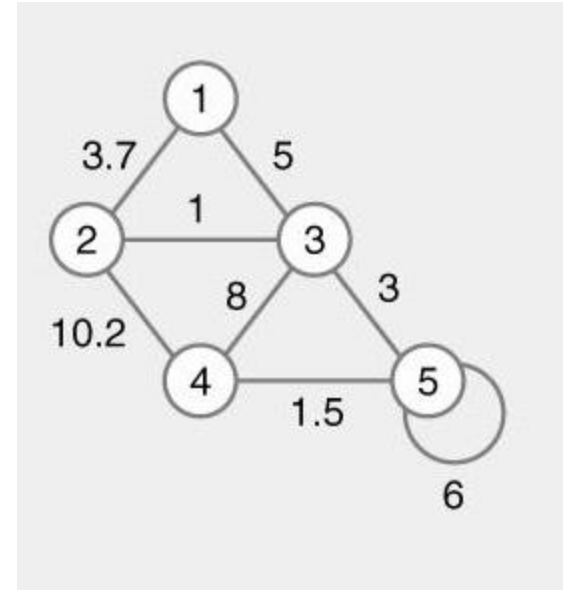
■ Example

■ Unweighted graph

```
node 1: {2, 3}
node 2: {1, 3, 4}
node 3: {1, 2, 4, 5}
node 4: {2, 3, 5}
node 5: {3, 4, 5}
```

■ Weighted graph

```
node 1: {2=3.7, 3=5}
node 2: {1=3.7, 3=1, 4=10.2}
node 3: {1=5, 2=1, 4=8, 5=3}
node 4: {2=10.2, 3=8, 5=1.5}
node 5: {3=3, 4=1.5, 5=6}
```



Adjacency Set / Map

■ Representation

- For each node, store
 - Set or map of neighbors / successors
- For unweighted graph
 - Use **set** of neighbors
- For weighted graph
 - Use **map** of neighbors, w/ value = weight of edge
- For undirected graph with edge ($a \leftrightarrow b$)
 - Nodes a & b need to store each other as neighbor
- For directed graph with edge ($a \rightarrow b$)
 - Node a needs to store node b as neighbor

Graph Space Requirements

■ Adjacency matrix

- $\frac{1}{2} N^2$ entries (for graph with N nodes, E edges)

- Many empty entries for large, sparse graphs

■ Adjacency list

- $2 \times E$ entries

■ Adjacency set / map

- $2 \times E$ entries

- Space overhead per entry

- Higher than for adjacency list

Graph Time Requirements

■ Adjacency matrix

- Can find individual edge (a,b) quickly
- Examine entry in array `Edge[a,b]`
 - Constant time operation

■ Adjacency list / set / map

- Can find all edges for node (a) quickly
- Iterate through collection of edges for a
 - On average E / N edges per node

Graph Time Requirements

■ Average Complexity of operations

■ For graph with N nodes, E edges

Operation	Adj Matrix	Adj List	Adj Set/Map
Find edge	$O(1)$	$O(E/N)$	$O(1)$
Insert edge	$O(1)$	$O(E/N)$	$O(1)$
Delete edge	$O(1)$	$O(E/N)$	$O(1)$
Enumerate edges for node	$O(N)$	$O(E/N)$	$O(E/N)$

Choosing Graph Implementations

- **Graph density**

- **Ratio edges to nodes (dense vs. sparse)**

- **Graph algorithm**

- **Neighbor based**

- For each node X in graph

- For each neighbor Y of X // adj list faster if sparse

- doWork()

- **Connection based**

- For each node X in ...

- For each node Y in ...

- if (X,Y) is an edge // adj matrix faster if dense

- doWork()