

CMSC 132: Object-Oriented Programming II

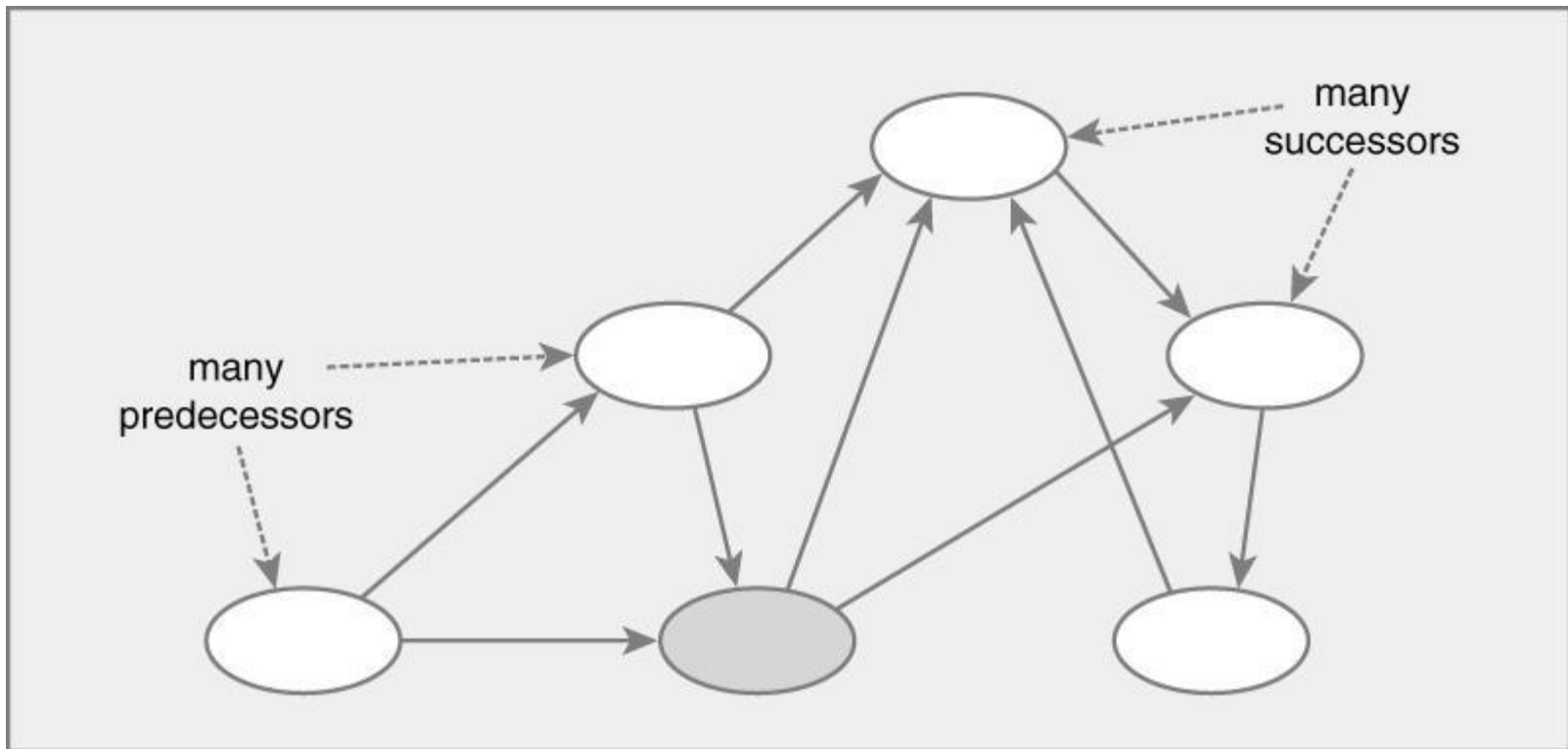


Graphs & Graph Traversal

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Graph Data Structures

- **Many-to-many relationship between elements**
 - Each element has **multiple predecessors**
 - Each element has **multiple successors**



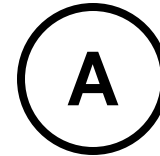
Graph Definitions

■ Node

- Element of graph

- State

- List of adjacent/neighbor/successor nodes



■ Edge

- Connection between two nodes

- State

- Endpoints of edge



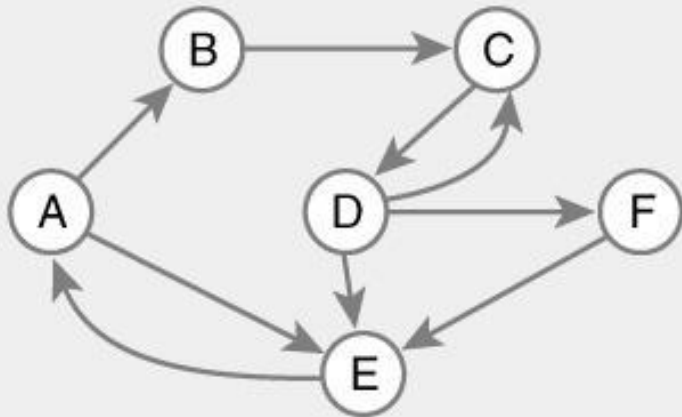
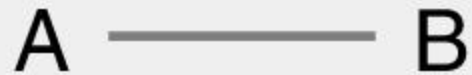
Graph Definitions

■ Directed graph

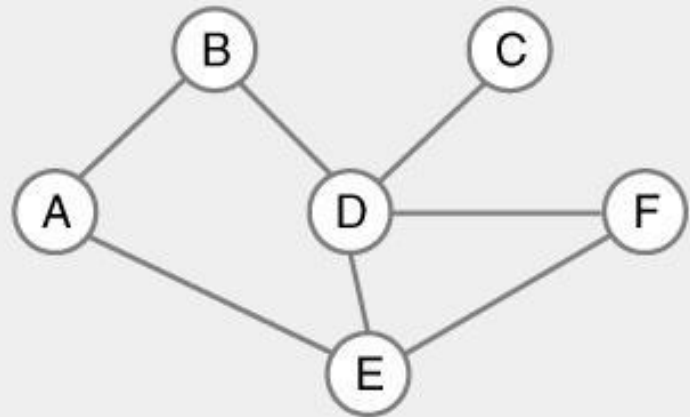
■ Directed edges

■ Undirected graph

■ Undirected edges



(a) Directed graph

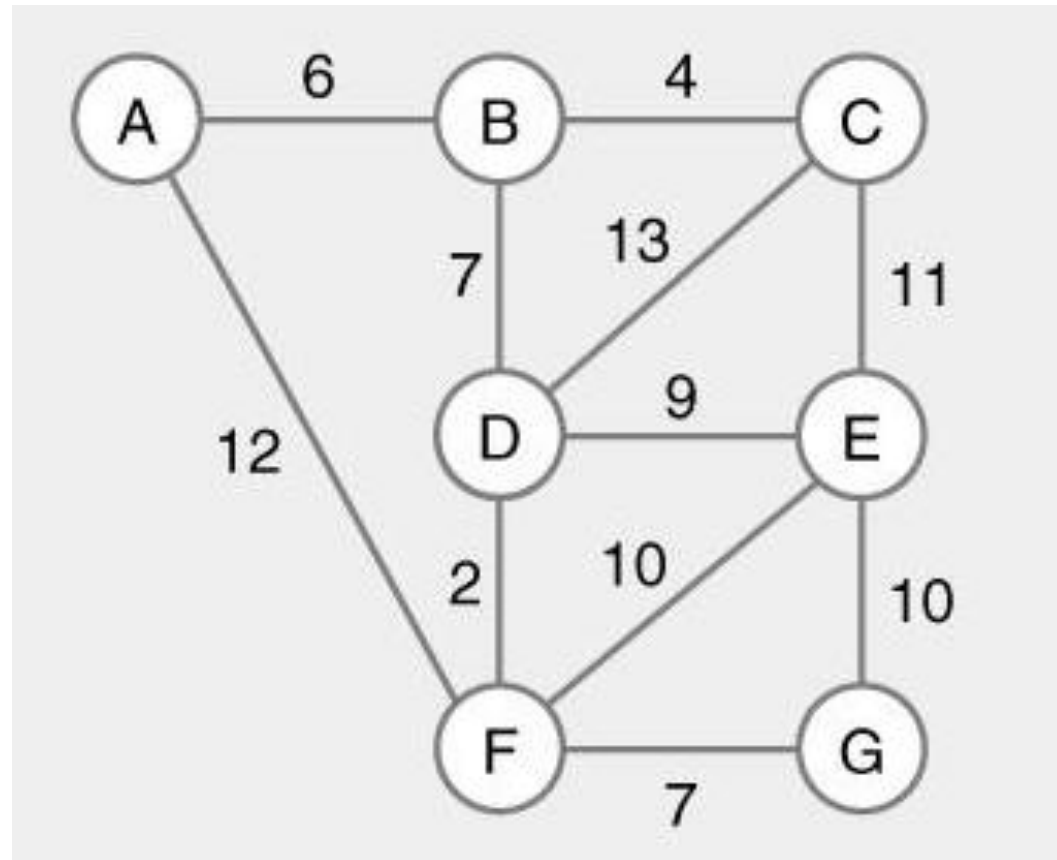


(b) Undirected graph

Graph Definitions

■ Weighted graph

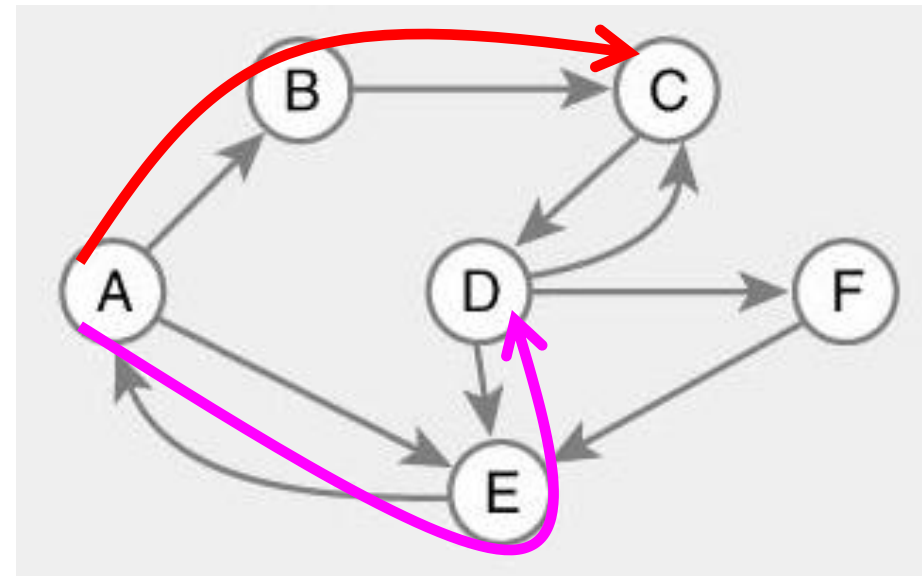
- Weight (cost) associated with each edge



Graph Definitions

■ Path

- Sequence of nodes n_1, n_2, \dots, n_k
- Edge exists between each pair of nodes n_i, n_{i+1}
- Example
 - A, B, C is a path
 - A, E, D is not a path



Graph Definitions

■ Cycle

- Path that ends back at starting node

- Example

 - A, E, A

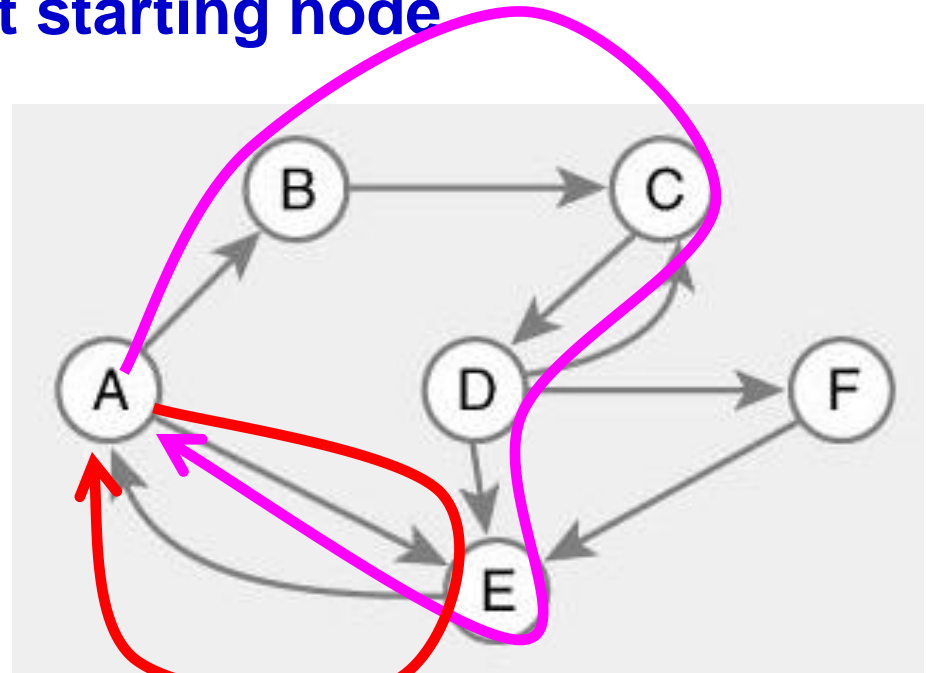
 - A, B, C, D, E, A

■ Simple path

- No cycles in path

■ Acyclic graph

- No cycles in graph



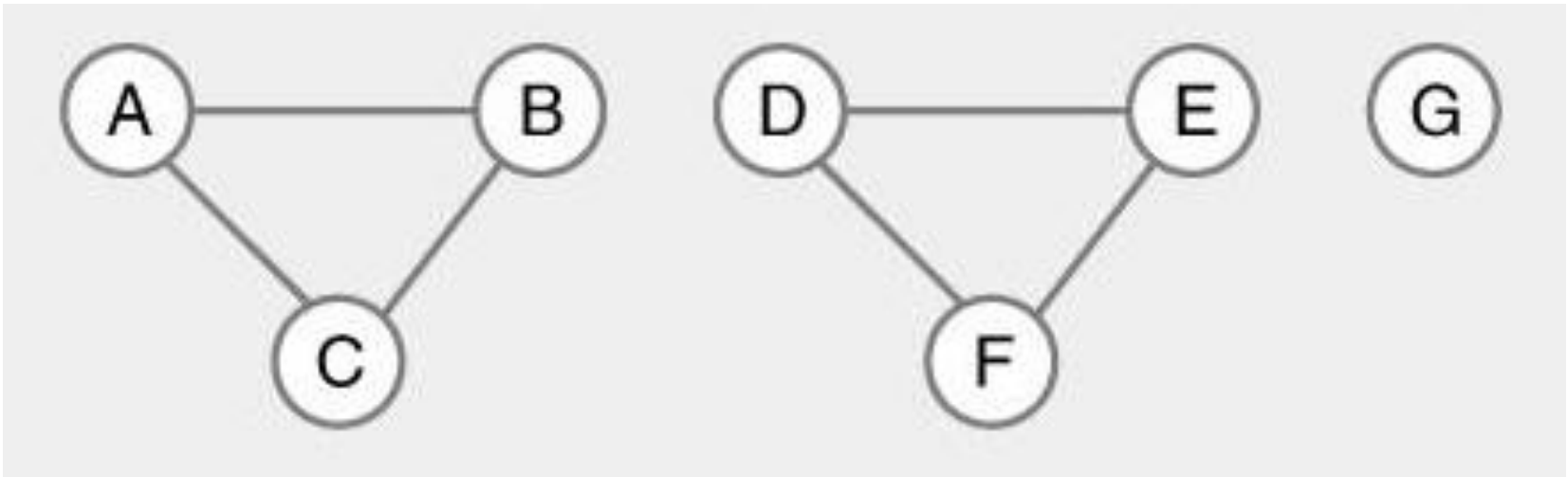
Graph Definitions

■ Connected Graph

- Every node in the graph is reachable from every other node in the graph

■ Unconnected graph

- Graph that has several disjoint components



Unconnected graph

Graph Operations

■ Traversal (search)

- Visit each node in graph exactly once
- Usually perform computation at each node
- Two approaches
 - Breadth first search (BFS)
 - Depth first search (DFS)

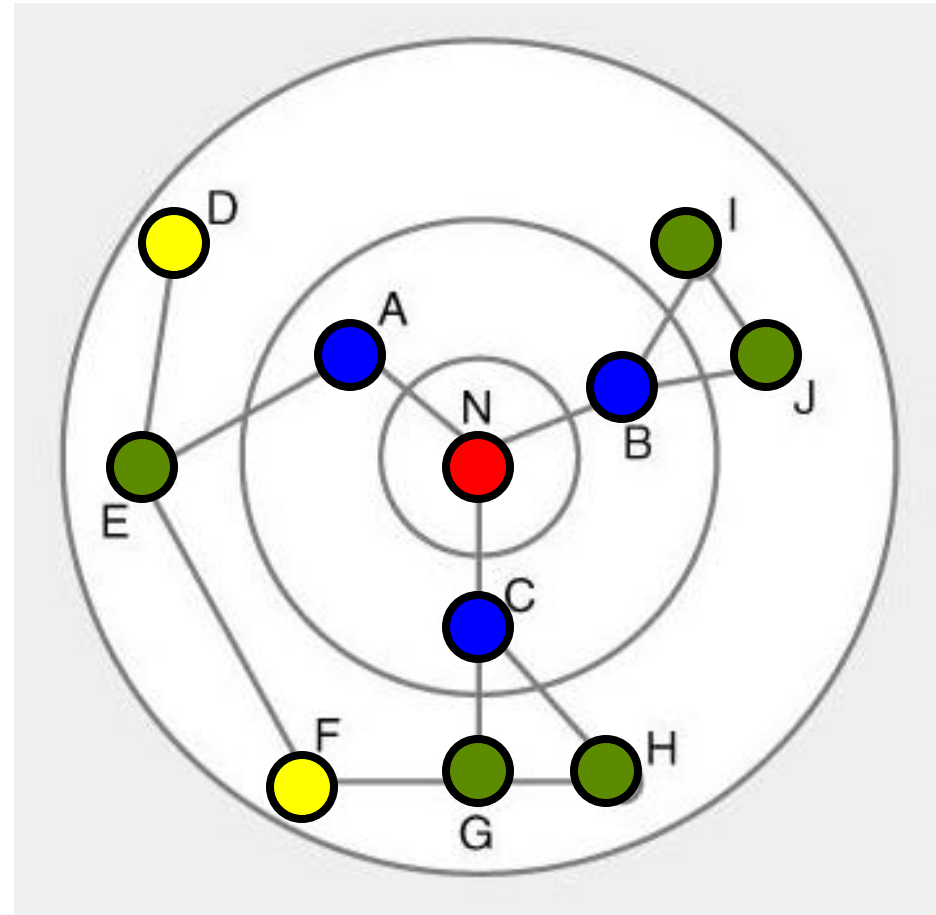
Breadth-first Search (BFS)

■ Approach

- Visit all neighbors of node first
- View as series of expanding circles
- Keep list of nodes to visit in queue

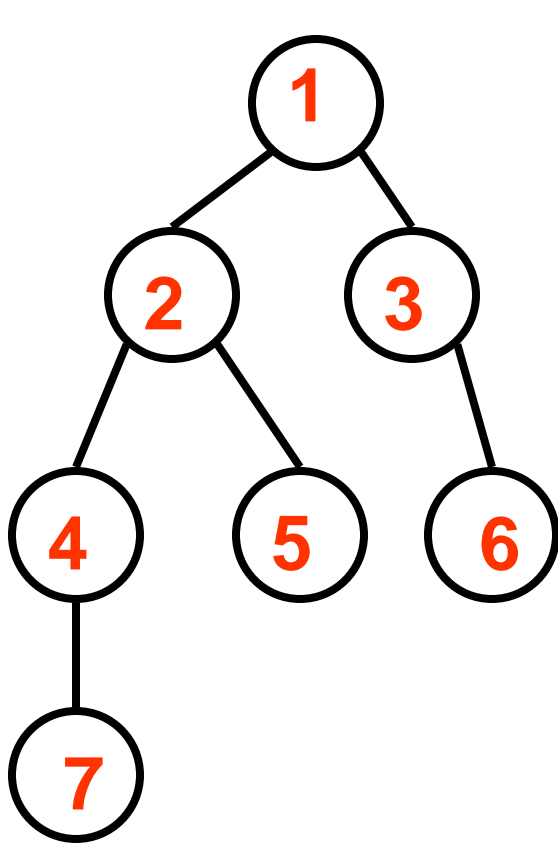
■ Example traversal

1. n
2. a, c, b
3. e, g, h, i, j
4. d, f

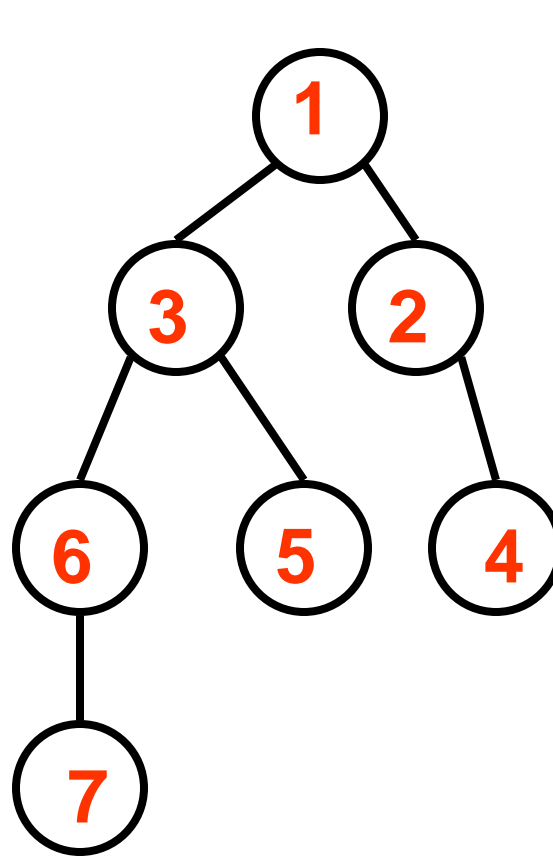


Breadth-first Tree Traversal

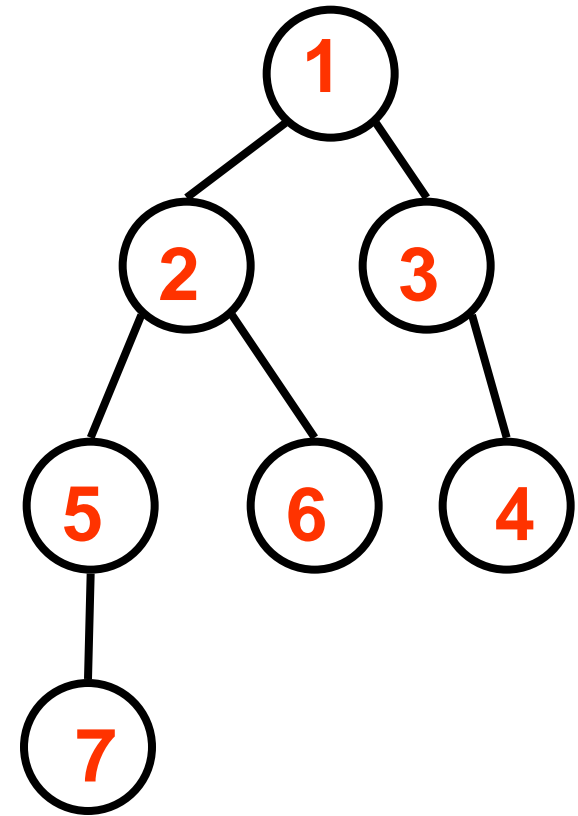
■ Example traversals starting from 1



Left to right



Right to left



Random

Traversals Orders

■ Order of successors

■ For tree

- Can order children nodes from left to right

■ For graph

- Left to right doesn't make much sense
- Each node just has a set of successors and predecessors; there is no order among edges

■ For breadth first search

- Visit all nodes at distance k from starting point
- Before visiting any nodes at (minimum) distance $k+1$ from starting point

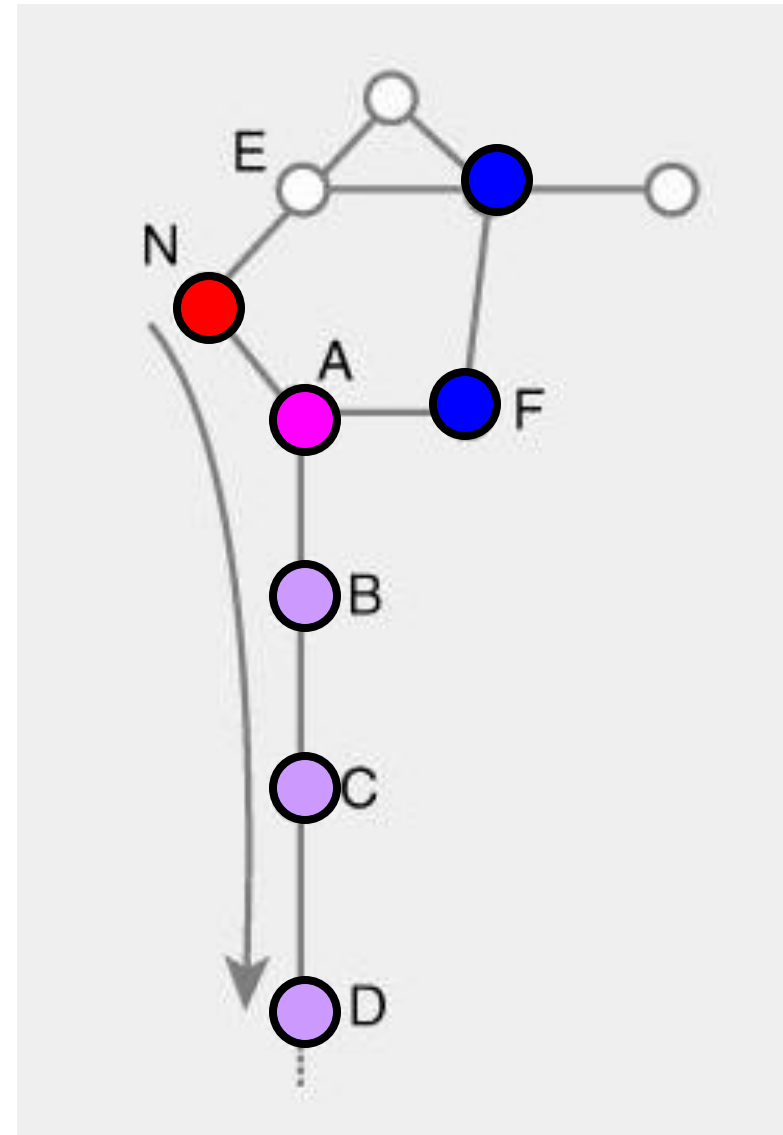
Depth-first Search (DFS)

■ Approach

- Visit all nodes on path first
- **Backtrack** when path ends
- Keep list of nodes to visit in a stack

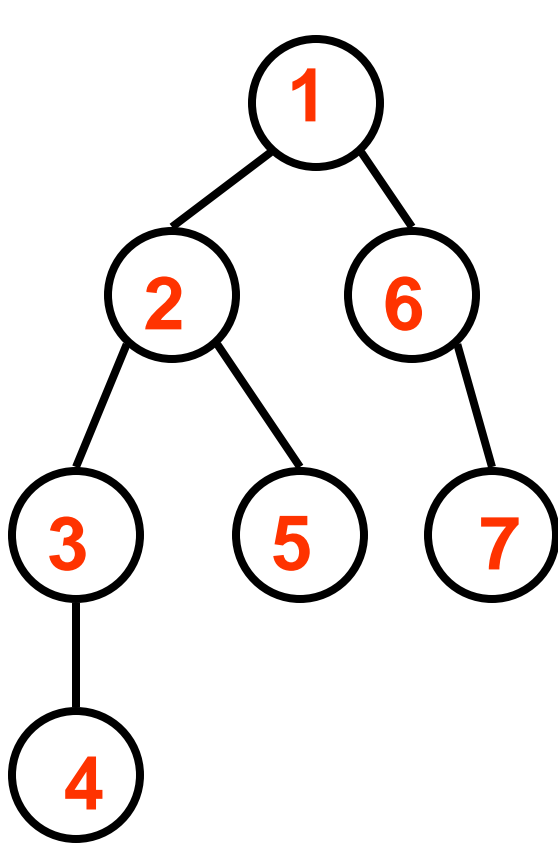
■ Example traversal

1. **N**
2. **A**
3. **B, C, D, ...**
4. **F...**

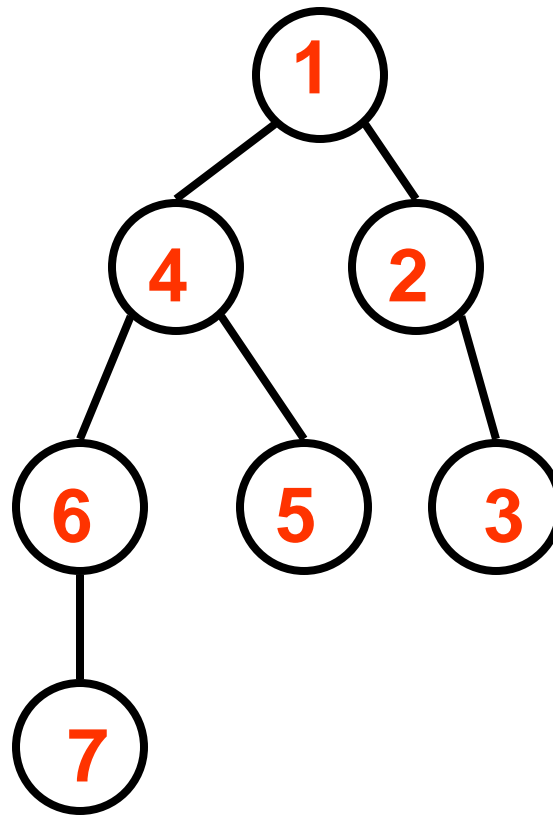


Depth-first Tree Traversal

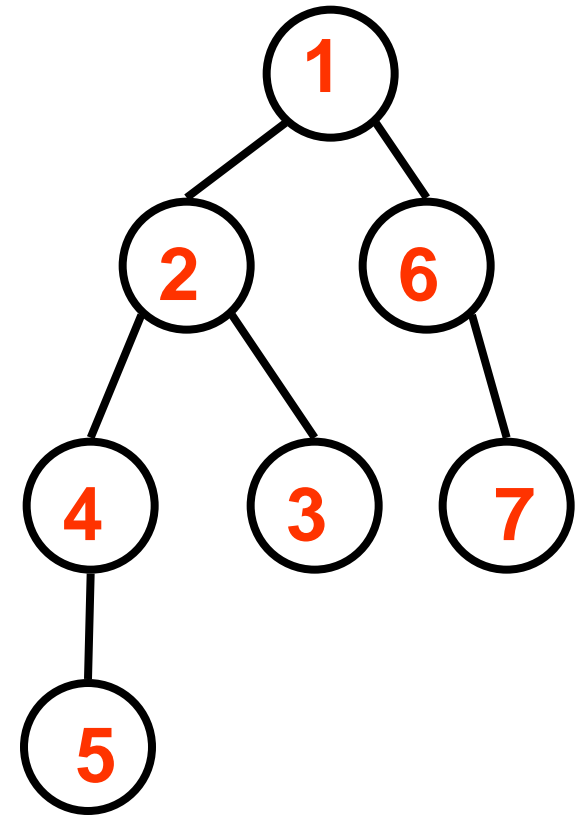
■ Example traversals from 1 (preorder)



Left to right



Right to left



Random

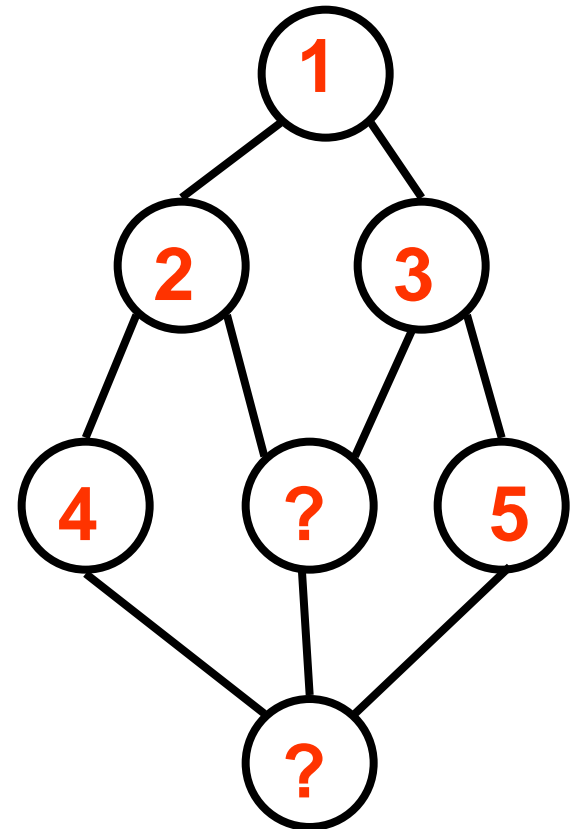
Traversal Algorithms

■ Issue

- How to avoid revisiting nodes
- Infinite loop if cycles present

■ Approaches

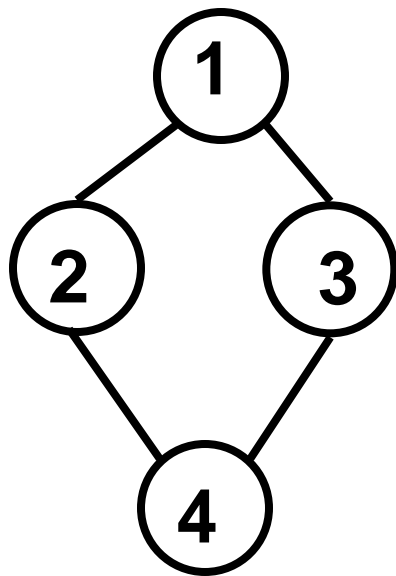
- Record set of visited nodes
- Mark nodes as visited



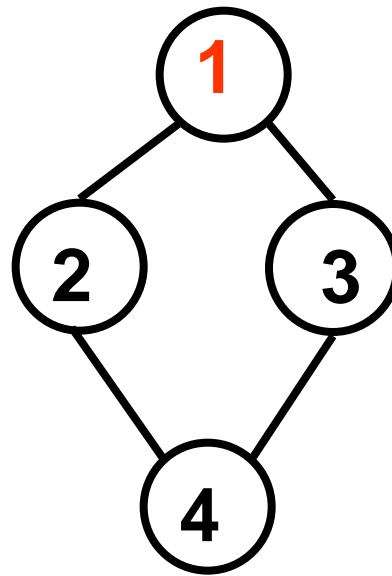
Traversal – Avoid Revisiting Nodes

■ Record set of visited nodes

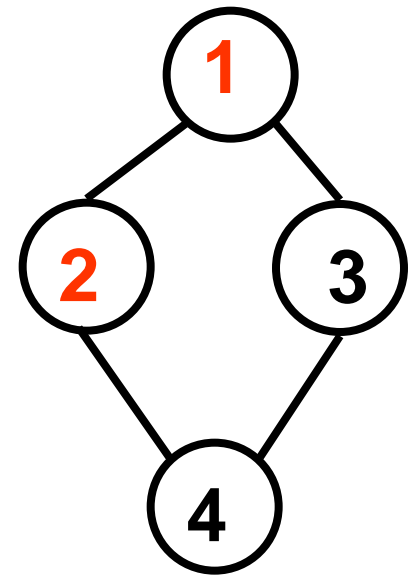
- Initialize { Visited } to empty set
- Add to { Visited } as nodes is visited
- Skip nodes already in { Visited }



$V = \emptyset$



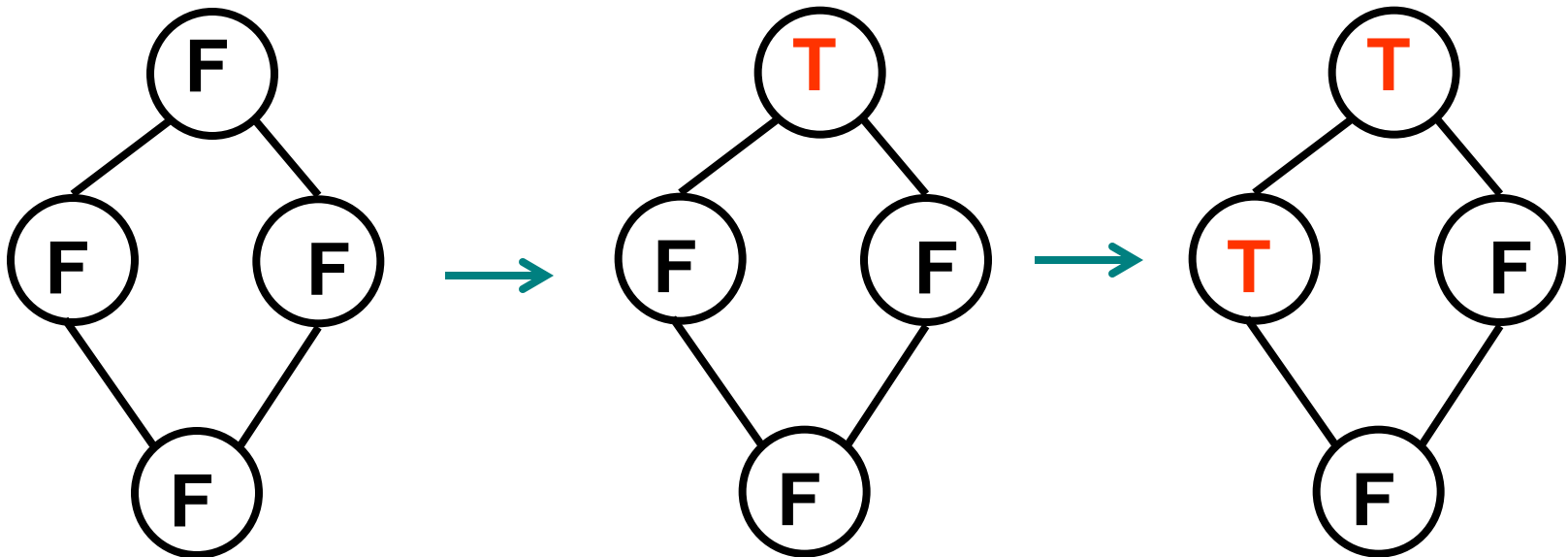
$V = \{ 1 \}$



$V = \{ 1, 2 \}$

Traversal – Avoid Revisiting Nodes

- **Mark nodes as visited**
 - Initialize tag on all nodes (to False)
 - Set tag (to True) as node is visited
 - Skip nodes with tag = True



Traversal Algorithm Using Sets

{ Visited } = \emptyset

{ Discovered } = { 1st node }

while ({ Discovered } $\neq \emptyset$)

take node X out of { Discovered }

if X not in { Visited }

add X to { Visited }

for each successor Y of X

if (Y is not in { Visited })

add Y to { Discovered }

Traversal Algorithm Using Tags

for all nodes X

set X.tag = False

{ Discovered } = { 1st node }

while ({ Discovered } $\neq \emptyset$)

take node X out of { Discovered }

if (X.tag = False)

set X.tag = True

for each successor Y of X

if (Y.tag = False)

add Y to { Discovered }

BFS vs. DFS Traversal

- Order nodes taken out of { Discovered } key
- Implement { Discovered } as Queue
 - First in, first out
 - Traverse nodes breadth first
- Implement { Discovered } as Stack
 - First in, last out
 - Traverse nodes depth first

BFS Traversal Algorithm

for all nodes X

X.tag = False

put 1st node in Queue

while (Queue not empty)

take node X out of Queue

if (X.tag = False)

set X.tag = True

for each successor Y of X

if (Y.tag = False)

put Y in Queue

DFS Traversal Algorithm

for all nodes X

X.tag = False

put 1st node in Stack

while (Stack not empty)

pop X off Stack

if (X.tag = False)

set X.tag = True

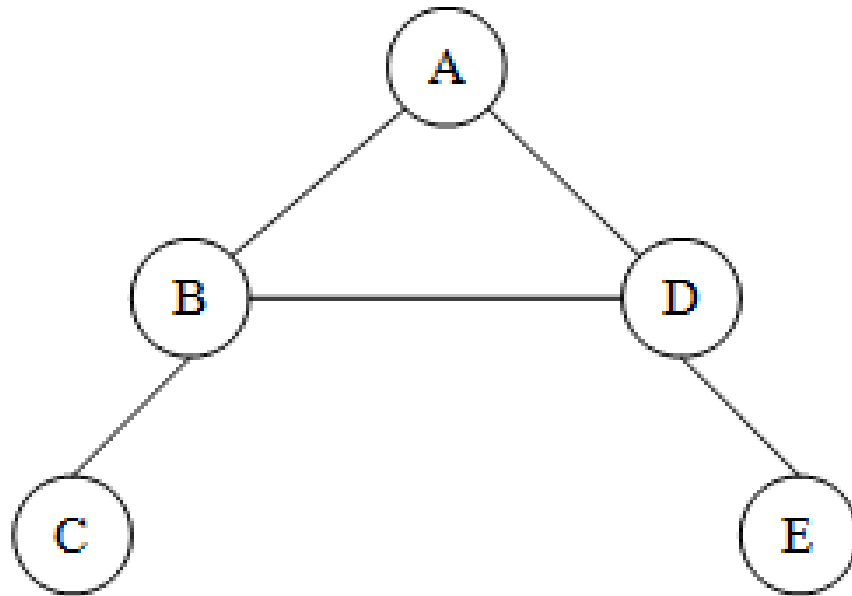
for each successor Y of X

if (Y.tag = False)

push Y onto Stack

Example

- Let's do a BFS/DFS using the following graph (start vertex A)



Recursive Graph Traversal

- Can traverse graph using recursive algorithm
 - Recursively visit successors
- Approach
 - Visit (X)
 - for each successor Y of X
 - Visit (Y)
- Implicit call stack & backtracking
 - Results in depth-first traversal

Recursive DFS Algorithm

Traverse()

for all nodes X

set X.tag = False

Visit (1st node)

Visit (X)

set X.tag = True

for each successor Y of X

if (Y.tag = False)

Visit (Y)