

# CMSC 330: Organization of Programming Languages

## Examples of REs & Finite Automata

## Describing Regular Expressions

- a)  $0(0|1)^*0$ 
  - All strings beginning and ending in 0
- b)  $((\epsilon|0)1^*)^*$ 
  - All strings
- c)  $(0|1)^*0(0|1)(0|1)$ 
  - All strings with 0 as third digit from right

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## Creating Regular Expressions

For all strings of 0's and 1's that...

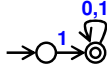
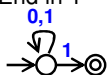
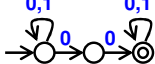
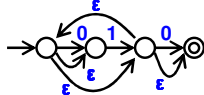
- a) Begin in 1
  - $1(0|1)^*$
- b) End in 1
  - $(0|1)^*1$
- c) Contains 00
  - $(0|1)^*00(0|1)^*$
- d) Do not contain 00
  - $(0|1)^*(\epsilon|0)$

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## Creating NFA

For all strings of 0's and 1's that...

- a) Begin in 1
 
- b) End in 1
 
- c) Contains 00
 
- d) Do not contain 00
 

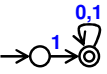
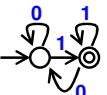
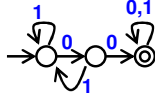
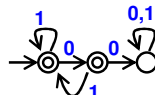
Based on regular expression

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## Creating DFA

For all strings of 0's and 1's that...

- a) Begin in 1
 
- b) End in 1
 
- c) Contains 00
 
- d) Do not contain 00
 

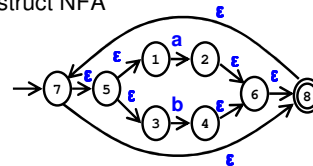
Swap final / non-final states!

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## For RE $(a | b)^*$

- a) Construct NFA



- b) Accept ababbab

7,5,1,2,6,8,7,5,3,4,6,8,7,5,1,2,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,1,2,6,8,7,5,3,4,6,8 accept

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## For RE $(a | b)^*$

### c) Reduce NFA to DFA

- Start =  $\epsilon$ -closure(7) = {7,5,1,3,8}
- $R = \{\{7,5,1,3,8\}\}$
- $r \in R = \{7,5,1,3,8\}$  // mark DFA state
- move ({7,5,1,3,8}, a) = {2}
  - $e = \epsilon$ -closure({2}) = {2,6,8,7,5,1,3} // new DFA state
  - $R = R \cup \{2,6,8,7,5,1,3\}$  // add to R
  - $\delta = \delta \cup (\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\})$
- move ({7,5,1,3,8}, b) = {4}
  - $e = \epsilon$ -closure({4}) = {4,6,8,7,5,1,3} // new DFA state
  - $R = R \cup \{4,6,8,7,5,1,3\}$  // add to R
  - $\delta = \delta \cup (\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\})$

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## For RE $(a | b)^*$

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r \in R = \{2,6,8,7,5,1,3\}$  // mark DFA state
- move ({2,6,8,7,5,1,3}, a) = {2}
  - $e = \epsilon$ -closure({2}) = {2,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})$
- move ({2,6,8,7,5,1,3}, b) = {4}
  - $e = \epsilon$ -closure({4}) = {4,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$

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## For RE $(a | b)^*$

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r \in R = \{4,6,8,7,5,1,3\}$  // mark DFA state
- move ({4,6,8,7,5,1,3}, a) = {2}
  - $e = \epsilon$ -closure({2}) = {2,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})$
- move ({4,6,8,7,5,1,3}, b) = {4}
  - $e = \epsilon$ -closure({4}) = {4,6,8,7,5,1,3} // existing DFA state
  - $\delta = \delta \cup (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$
- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$ 
  - No more unmarked states to process
- $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$ 
  - Since  $8 \in F_n$

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## For RE $(a | b)^*$

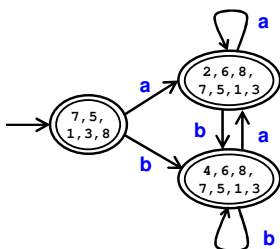
- Resulting DFA
  - $\Sigma = \{a,b\}$
  - $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
  - $r_0 = \{7,5,1,3,8\}$
  - $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
  - $\delta = \{ (\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\}), (\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\}), (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}), (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}), (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}), (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}) \}$

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## For RE $(a | b)^*$

- NFA to DFA reduction pictorial



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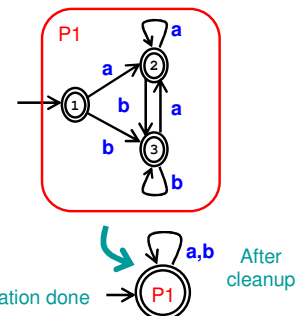
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## For RE $(a | b)^*$

### d) Minimize DFA

- Initial partitions
  - Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
  - Reject  $\rightarrow \emptyset$
- Split partition?
  - move(1,a)  $\rightarrow P1$
  - move(2,a)  $\rightarrow P1$
  - move(3,a)  $\rightarrow P1$
  - move(1,b)  $\rightarrow P1$
  - move(2,b)  $\rightarrow P1$
  - move(3,b)  $\rightarrow P1$

Not required, minimization done

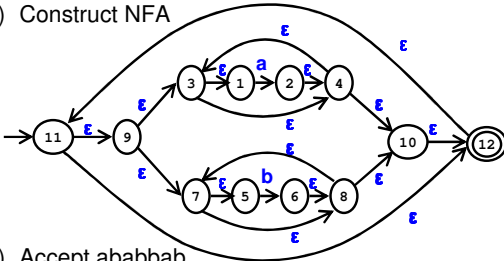


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### For RE $(a^* | b^*)^*$

a) Construct NFA



b) Accept ababbab

11,9,3,1,2,4,10,12,11,9,7,5,6,8,10,12,11,9,3,1,2,4,10,12...

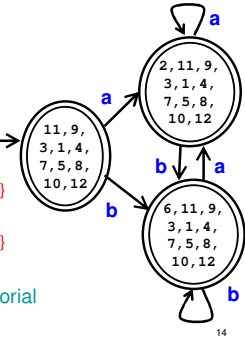
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### For RE $(a^* | b^*)^*$

c) Reduce NFA to DFA

- Start =  $\epsilon$ -closure(1)
- = {11,9,3,1,4,7,5,8,10,12}
- $R = \{\{11,9...12\}\}$
- $r \in R = \{11,9...12\}$
- move ( $\{11,9...12\}, a$ ) = 2
- $\epsilon$ -closure( $\{2\}$ ) = {2,11,9...12}
- move ( $\{11,9,...,12\}, b$ ) = 6
- $\epsilon$ -closure( $\{6\}$ ) = {6,11,9...12}
- ...
- NFA to DFA reduction pictorial



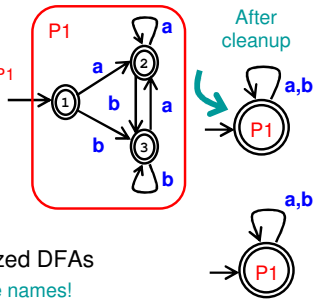
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### For RE $(a^* | b^*)^*$

d) Minimize DFA

- Initial partitions
- Accept  $\rightarrow \{1,2,3\} \rightarrow P1$
- Reject  $\rightarrow \emptyset$
- Split partition?
- move(1,a)  $\rightarrow P1$
- ...
- Not required, minimization done



e) Compare 2 minimized DFAs

- Identical up to state names!

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