CMSC 330: Organization of Programming Languages

Context-Free Grammars

Motivation

- Programs are just strings of text
  - But they're strings that have a certain structure
- Informal description of syntax of a C program
  - A C program is a list of declarations and definitions
  - A function definition contains parameters and a body
  - A function body is a sequence of statements
  - A statement is an expression, if, goto, etc.
  - An expression may be assignment, addition, subtraction, etc

Motivation (cont'd)

- We want to describe program structure precisely
- Regular expressions are not enough
  - No regular expression for balanced pairs of ( )'s
    - {"()", "(())", "()()", ... } is not a regular language
- Instead, we'll use context-free grammars
  - These are *almost* enough for C, C++, Java

Context Free Grammar (CFG)

- A way of generating sets of strings or languages
- Grammar: $S \to 0S \mid 1S \mid \varepsilon$
  - Means every S may be replaced by 0S, 1S, or ε
  - Example
    - S ⇒ 0S // using $S \to 0S$
      ⇒ 01S // using $S \to 1S$
      ⇒ 011S // using $S \to 1S$
      ⇒ 011 // using $S \to \varepsilon$
- Grammar is same as regular expression (0|1)*
  - Generates / accepts the same set of strings

Context-Free Grammars (CFGs)

- But CFGs can do a lot more!
  - $S \to (S) \mid \varepsilon$ // generates balanced pairs of ( )'s
- In fact, CFGs subsume REs, DFAs, NFAs
  - There is a CFG that generates any regular language
  - But REs are a better notation for regular languages
- CFGs can specify programming language syntax
  - CFGs (mostly) describe the parsing process

Formal Definition

- A context-free grammar G is a 4-tuple:
  - Σ – a finite set of *terminal* or *alphabet* symbols
    - Often written in lowercase
  - N – a finite, nonempty set of *nonterminal* symbols
    - Often written in uppercase
    - It must be that $N \cap \Sigma = \varnothing$
  - P – a set of *productions* of the form $N \to (\Sigma|N)^*$
    - Informally this means that the nonterminal can be replaced by the string of zero or more terminals or nonterminals to the right of the →
    - Can think of productions as rewriting rules
  - $S \in N$ – the *start symbol*

Backus-Naur Form

- Context-free grammar production rules are also called Backus-Naur Form or BNF
  - A production like A → B c D is written in BNF as <A> ::= <B> c <D> (Non-terminals written with angle brackets and ::= instead of →)
  - Often used to describe language syntax
- BNF was designed by
  - John Backus
    - Chair of the Algol committee in the early 1960s
  - Peter Naur
    - Secretary of the committee, who used this notation to describe Algol in 1962

Informal Definition of Acceptance

- A string is accepted by a CFG if there is
  - Some sequence of applying productions (rewrites) starting at the start symbol that generates the string
- Example
  - Grammar: S → 0S | 1S | ε
  - Sequence generating the string 010
    - S ⇒ 0S ⇒ 01S ⇒ 010S ⇒ 010
- Terminology
  - Such a sequence of rewrites is a derivation or parse
  - Discovering the derivation is called parsing

Derivations

- Notation
  - ⇒ indicates a derivation of one step
  - ⇒+ indicates a derivation of one or more steps
  - ⇒* indicates a derivation of zero or more steps
- Example
  - S → 0S | 1S | ε
- For the string 010
  - S ⇒ 0S ⇒ 01S ⇒ 010S ⇒ 010
  - S ⇒+ 010
  - S ⇒* S

Practice

- Try to make a grammar which accepts
  - 0*|1*
    - S → A | B
    - A → 0A | ε
    - B → 1B | ε
  - 0^n1^n where n ≥ 0
    - S → 0S1 | ε
  - 0^n1^m where m ≤ n
    - S → 0S1 | 0S ε
- Give some example strings from this language
  - S → 0 | 1S
    - 0, 10, 110, 1110, 11110, …
  - What language is it?
    - 1*0

Example

S → aS | T
T → bT | U
U → cU | ε

- A derivation:
  - S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac
    - Abbreviated as S ⇒+ ac
  - S ⇒ T ⇒ U ⇒ ε
- Is there any derivation
  - S ⇒+ ccc ?   S ⇒+ Sa ?
  - S ⇒+ bab ?   S ⇒+ bU ?

Example (cont'd)

S → aS | T
T → bT | U
U → cU | ε

- Generates what language?
- Do other grammars generate this language?

S → ABC
A → aA | ε
B → bB | ε
C → cC | ε

  - So grammars are not unique

Example: Arithmetic Expressions (Limited)

- E → a | b | c | E+E | E-E | E*E | (E)
  - An expression E is either a letter a, b, or c
  - Or an E followed by + followed by an E
  - etc.
- This describes or generates a set of strings
  - {a, b, c, a+b, a+a, a*c, a-(b*a), c*(b + d)}
- Example strings not in the language
  - d, c(a), a+, b**c, etc.

Example, Formally

- Formally, the grammar we just showed is

  Σ = { +, -, *, (,), a, b, c } // terminals
  N = { E } // nonterminals
  P = { E → a, E → b, E → c, // productions
  E → E-E, E → E+E,
  E → E*E, E → (E) }
  S = E // start symbol

Uniqueness of Grammars

- Grammars are not unique
  - Different grammars can generate same set of strings
- Following grammar generates the same set of strings as the previous grammar:

  E → E+T | E-T | T
  T → T*P | P
  P → (E) | a | b | c

Notational Shortcuts

- A production is of the form
  - left-hand side (LHS) → right hand side (RHS)
- If not specified
  - Assume LHS of first listed production is the start symbol
- Productions with the same LHS
  - Are usually combined with |
- If a production has an empty RHS
  - It means the RHS is ε

S → ABC // S is start symb
A → aA
| b // A → b
| // A → ε

Sentential Forms and Derivations

- A sentential form is a string of terminals and nonterminals produced from that start symbol
- Inductively
  - The start symbol is a sentential form for a grammar
  - If αAδ is a sentential form for a grammar, where (α and δ ∈ (N|Σ)*), and A → γ is a production, then αγδ is a sentential form for the grammar
    - In this case, we say that αAδ *derives* αγδ in one step, which is written as αAδ ⇒ αγδ

Sentential Forms Example

- Given grammar
  - S → 0S | 1S | ε
- Possible derivations
  - S ⇒ 0S ⇒ 01S ⇒ 010S ⇒ 010
    - S, 0S, 01S, 010S, 010 are sentential forms
  - S ⇒ 1S ⇒ 11S ⇒ 111S ⇒ 111
    - S, 1S, 11S, 111S, 111 are sentential forms
  - S ⇒ ε
    - S, ε are sentential forms
- In other words
  - If S ⇒* α, then α is a sentential form

The Language Generated by a CFG

- The *language generated by a grammar G* is

$$L(G) = \{ \omega \mid \omega \in \Sigma^* \text{ and } S \Rightarrow^+ \omega \}$$

  - S is the start symbol of the grammar
  - Σ is the alphabet for that grammar
- In other words
  - All sentential forms with only terminals
  - All strings over Σ that can be derived from the start symbol via one or more productions

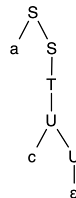
Parse Trees

- A *parse tree* shows how a string is produced by a grammar
  - Root node is the start symbol
  - Each interior node is a nonterminal
  - Children of node are symbols on r.h.s of production applied to that nonterminal
  - Leaves are all terminal symbols
- Reading the leaves left-to-right shows the string corresponding to the tree

Example

S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac

S → aS | T
T → bT | U
U → cU | ε



Leftmost and Rightmost Derivation

- Leftmost derivation
  - Leftmost nonterminal is replaced in each step
- Rightmost derivation
  - Rightmost nonterminal is replaced in each step
- Example
  - Grammar
    - S → AB, A → a, B → b
  - Leftmost derivation for "ab"
    - S ⇒ AB ⇒ aB ⇒ ab
  - Rightmost derivation for "ab"
    - S ⇒ AB ⇒ Ab ⇒ ab

Parse Tree For Derivations

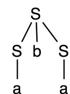
- Parse tree may be same for both leftmost & rightmost derivations
  - Example Grammar: S → a | SbS String: aba

    Leftmost Derivation

    S ⇒ SbS ⇒ abS ⇒ aba

    Rightmost Derivation

    S ⇒ SbS ⇒ Sba ⇒ aba



  - Parse trees don't show order productions are applied
- Every parse tree has a unique leftmost and a unique rightmost derivation

Parse Tree For Derivations (cont.)

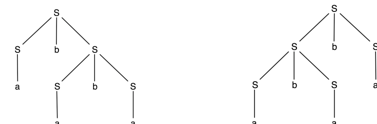
- Not every string has a unique parse tree
  - Example Grammar: S → a | SbS String: ababa

    Leftmost derivation

    S ⇒ SbS ⇒ abS ⇒ abSbS ⇒ ababS ⇒ ababa

    Another leftmost derivation

    S ⇒ SbS ⇒ SbSbS ⇒ abSbS ⇒ ababS ⇒ ababa



Ambiguity

- A grammar is ambiguous if a string may have multiple leftmost (or rightmost) derivations
  - Equivalent to multiple parse trees
  - Can be hard to determine

1. S → aS | T
   T → bT | U   **No**
   U → cU | ε

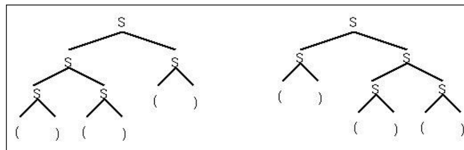
2. S → SS | () | (S)   **?**

Ambiguity (cont'd)

- 2 different parse trees for the same string: ()()()
- 2 distinct leftmost derivations :

  S ⇒ SS ⇒ SSS ⇒()SS ⇒()()S ⇒()()()

  S ⇒ SS ⇒ ()S ⇒()SS ⇒()()S ⇒()()()



More on Leftmost/Rightmost Derivations

- Is the following derivation leftmost or rightmost?

  S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac

  – Both! At most one non-terminal in each sentential form, so there's no choice which non-terminals to expand

- How about the following derivation?
  – S ⇒ SbS ⇒ SbSbS ⇒ SbabS ⇒ ababS ⇒ ababa

  Neither! Selects left, center, left, and rightmost nonterminals

Tips for Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

   A → xA | ε   Zero or more x's

   A → yA | y   One or more y's

2. Use separate nonterminals to generate disjoint parts of a language, and then combine in a production

   G = S → AB

   A → aA | ε

   B → bB | ε

   L(G) = a^*b^*

Tips for Designing Grammars (cont'd)

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

   $\{a^nb^n \mid n \geq 0\}$ (not a regular language!)

   S → aSb | ε

   Example: S ⇒ aSb ⇒ aaSbb ⇒ aabb

   $\{a^nb^{2n} \mid n \geq 0\}$

   S → aSbb | ε

Tips for Designing Grammars (cont'd)

$\{a^nb^m \mid m \geq 2n, n \geq 0\}$

S → aSbb | B | ε

B → bB | b

The following grammar also works:

S → aSbb | B

B → bB | ε

How about the following?

S → aSbb | bS | ε

Tips for Designing Grammars (cont'd)

$\{a^n b^m a^{n+m} \mid n \geq 0, m \geq 0\}$

Rewrite as $a^n b^m a^m a^n$, which now has matching superscripts (two pairs)

Would this grammar work?

S → aSa | B
B → bBa | ba

Corrected:

S → aSa | B
B → bBa | ε

The outer $a^n a^n$ are generated first, then the inner $b^m a^m$

Tips for Designing Grammars (cont'd)

4. For a language that's the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

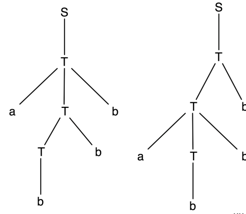
$\{ a^n b^m \mid m > n \geq 0\} \cup$
$\{ a^n c^m \mid m > n \geq 0\}$

Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

S → T | U
T → aTb | Tb | b — T generates the first set
U → aUc | Uc | c — U generates the second set

- What about the string abbb?
  - Ambiguous!



Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

Will this fix the ambiguity?

S → T | U
T → aTb | bT | b
U → aUc | cU | c

- It's not ambiguous, but it can generate invalid strings such as babb

Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

Unambiguous version

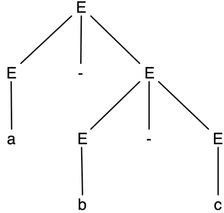
S → T | V
T → aTb | U
U → Ub | b
V → aVc | W
W → Wc | c

CFGs for Languages

- Recall that our goal is to describe programming languages with CFGs
- We had the following example which describes limited arithmetic expressions
  E → a | b | c | E+E | E-E | E*E | (E)
- What's wrong with using this grammar?
  – It's ambiguous!

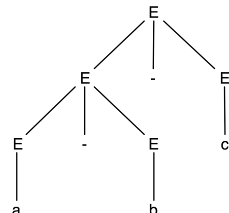
Example: a-b-c

E ⇒ E-E ⇒ a-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to a-(b-c)

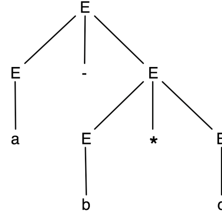
E ⇒ E-E ⇒ E-E-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to (a-b)-c

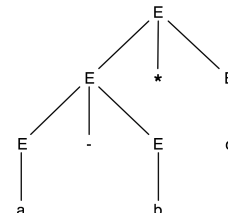
Example: a-b*c

E ⇒ E-E ⇒ a-E ⇒ a-E*E ⇒ a-b*E ⇒ a-b*c



Corresponds to a-(b*c)

E ⇒ E-E ⇒ E-E*E ⇒ a-E*E ⇒ a-b*E ⇒ a-b*c



Corresponds to (a-b)*c

Another Example: If-Then-Else

<stmt> ::= <assignment> | <if-stmt> | ...
<if-stmt> ::= if (<expr>) <stmt> |
if (<expr>) <stmt> else <stmt>

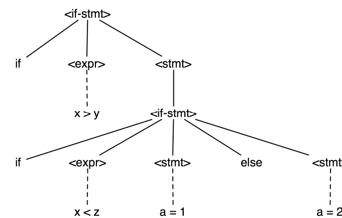
– (Here <>'s are used to denote nonterminals and ::= for productions)

- Consider the following program fragment:

```
if (x > y)
 if (x < z)
  a = 1;
 else a = 2;
```

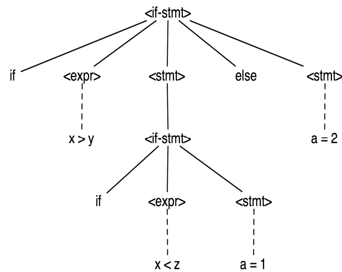
– Note: Ignore newlines

Parse Tree #1



- Else belongs to inner if

Parse Tree #2



- Else belongs to outer if

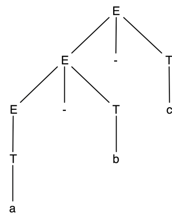
Dealing With Ambiguous Grammars

- Ambiguity is bad
  - Syntax is correct
  - But semantics differ depending on choice
    - Different associativity (a-b)-c vs. a-(b-c)
    - Different precedence (a-b)*c vs. a-(b*c)
    - Different control flow if (if else) vs. if (if) else
- Two approaches
  - Rewrite grammar
  - Use special parsing rules
    - Depending on parsing method (learn in CMSC 430)

Fixing the Expression Grammar

- Idea: Require that the right operand of all of the operators not have an operator it in, unless it's parenthesized

  E → E+T | E-T | E*T | T
  T → a | b | c | (E)



- Now only one parse tree for a-b-c
  - Left associative
  - Exercise: Give a derivation for the string a-(b-c)

What if We Wanted Right-Associativity?

- Left-recursive productions are used for left-associative operators
- Right-recursive productions are used for right-associative operators
- Left:

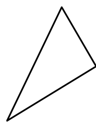
  E → E+T | E-T | E*T | T
  T → a | b | c | (E)
- Right:

  E → T+E | T-E | T*E | T
  T → a | b | c | (E)

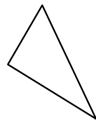
Parse Tree Shape

- The kind of recursion/associativity determines the shape of the parse tree

left recursion



right recursion

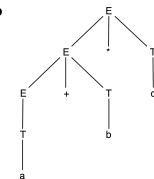


  - Exercise: draw a parse tree for a-b-c in the prior grammar in which subtraction is right-associative

A Different Problem

- How about the string a+b*c ?

  E → E+T | E-T | E*T | T
  T → a | b | c | (E)



- Doesn't have correct precedence for *
  - When a nonterminal has productions for several operators, they effectively have the same precedence
- How can we fix this?

Final Expression Grammar

E → E+T | E-T | T lowest precedence operators
T → T*P | P higher precedence
P → a | b | c | (E) highest precedence (parentheses)

- Exercises:
  - Construct tree and left and and right derivations for
    - a+b*c a*(b+c) a*b+c a-b-c
  - See what happens if you change the last set of productions to P → a | b | c | E | (E)
  - See what happens if you change the first set of productions to E → E +T | E-T | T | P

Regular expressions and CFGs

	Description	Machine
regular languages	regular expressions	DFAs, NFAs
context-free languages	context-free grammars	pushdown automata (PDAs)

- Programming languages are neither regular nor context-free
  - Usually almost context-free, with some hacks

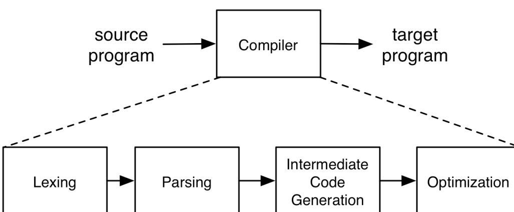
Pushdown Automaton (PDA)

- A pushdown automaton (PDA) is an abstract machine similar to the DFA
  - Has a finite set of states
  - Also has a *pushdown stack*
- Moves of the PDA are as follows:
  - An input symbol is read and the top symbol on the stack is read
  - Based on both inputs, the machine
    - Enters a new state, and
    - Writes zero or more symbols onto the pushdown stack
  - String accepted if the stack is empty at end of string

Power of PDAs

- PDAs are more powerful than DFAs
  - $a^n b^n$, which cannot be recognized by a DFA, can easily be recognized by the PDA
    - Stack all a symbols and, for each b, pop an a off the stack.
    - If the end of input is reached at the same time that the stack becomes empty, the string is accepted
- As with NFA, we can also have a NDPDA
  - NDPDA are more powerful than DPDA
  - NDPDA can recognize even length palindromes over {0,1}*, but a DPDA cannot. Why? (Hint: Consider palindromes over $\{0,1\}^* 2 \{0,1\}^*$)
- It is true, but less clear, that the languages accepted by NDPDAs are equivalent to the context-free languages

Steps of Compilation



Parsing

- There are many efficient techniques for turning strings into parse trees or ASTs
  - They all have strange names, like LL(k), SLR(k), LR(k)...
  - Take CMSC 430 for more details
- We will look at one very simple technique: *recursive descent parsing*
  - This is a “top-down” parsing algorithm because we’re going to begin at the start symbol and try to produce the string

Recursive Descent Parsing

- Goal
  - Determine if we can produce the string to be parsed from the grammar's start symbol
- Approach
  - Recursively replace nonterminal with RHS of production
- At each step, we'll keep track of two facts
  - What tree node are we trying to match?
  - What is the lookahead (next token of the input string)?
    - Helps guide selection of production used to replace nonterminal

Recursive Descent Parsing (cont.)

- At each step, 3 possible cases
  - If we’re trying to match a terminal
    - If the lookahead is that token, then succeed, advance the lookahead, and continue
  - If we’re trying to match a nonterminal
    - Pick which production to apply based on the lookahead
  - Otherwise fail with a parsing error

Example

$E \rightarrow id = n \mid \{ L \}$

$L \rightarrow E ; L \mid \varepsilon$

– Here n is an integer and id is an identifier

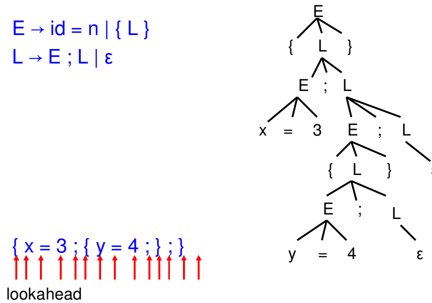
- One input might be
  – { x = 3; { y = 4; }; }
  – This would get turned into a list of tokens

    `{ x = 3 ; { y = 4 ; } ; }`
  – And we want to turn it into a parse tree

Example (cont'd)

$E \rightarrow id = n \mid \{ L \}$

$L \rightarrow E ; L \mid \varepsilon$



Recursive Descent Parsing (cont.)

- Key step
  – Choosing which production should be selected
- Two approaches
  – Backtracking
    - Choose some production
    - If fails, try different production
    - Parse fails if all choices fail
  – Predictive parsing
    - Analyze grammar to find FIRST sets for productions
    - Compare with lookahead to decide which production to select
    - Parse fails if lookahead does not match FIRST

First Sets

- Motivating example
  – The lookahead is x
  – Given grammar $S \rightarrow xyz \mid abc$
    - Select $S \rightarrow xyz$ since 1st terminal in RHS matches x
  – Given grammar $S \rightarrow A \mid B$ $A \rightarrow x \mid y$ $B \rightarrow z$
    - Select $S \rightarrow A$, since A can derive string beginning with x
- In general
  – Choose a production that can derive a sentential form beginning with the lookahead
  – Need to know what terminal may be first in any sentential form derived from a nonterminal / production

First Sets

- Definition
  – First(γ), for any terminal or nonterminal γ, is the set of initial terminals of all strings that γ may expand to
  – We'll use this to decide what production to apply
- Examples
  – Given grammar $S \rightarrow xyz \mid abc$
    - First(xyz) = { x }, First(abc) = { a }
    - First(S) = First(xyz) U First(abc) = { x, a }
  – Given grammar $S \rightarrow A \mid B$ $A \rightarrow x \mid y$ $B \rightarrow z$
    - First(x) = { x }, First(y) = { y }, First(A) = { x, y }
    - First(z) = { z }, First(B) = { z }
    - First(S) = { x, y, z }

Calculating First(γ)

- For terminal a, First(a) = { a }
- For a nonterminal N:
  – If $N \rightarrow \varepsilon$, then add ε to First(N)
  – If $N \rightarrow \alpha_1 \alpha_2 \ldots \alpha_n$, then (note the α_i are all the symbols on the right side of one single production):
    - Add $First(\alpha_1 \alpha_2 \ldots \alpha_n)$ to First(N), where $First(\alpha_1 \alpha_2 \ldots \alpha_n)$ is defined as
      – $First(\alpha_1)$ if $\varepsilon \notin First(\alpha_1)$
      – Otherwise $(First(\alpha_1) - \varepsilon) \cup First(\alpha_2 \ldots \alpha_n)$
    - If $\varepsilon \in First(\alpha_i)$ for all i, $1 \leq i \leq k$, then add ε to First(N)

Examples

$E \rightarrow id = n \mid \{ L \}$
$L \rightarrow E ; L \mid \varepsilon$

First(id) = { id }
First("=") = { "=" }
First(n) = { n }
First("{")= { "{" }
First("}")= { "}" }
First(";")= { ";" }
First(E) = { id, "{" }
First(L) = { id, "{", ε }

$E \rightarrow id = n \mid \{ L \} \mid \varepsilon$
$L \rightarrow E ; L \mid \varepsilon$

First(id) = { id }
First("=") = { "=" }
First(n) = { n }
First("{")= { "{" }
First("}")= { "}" }
First(";")= { ";" }
First(E) = { id, "{", ε }
First(L) = { id, "{", ";", ε }

Recursive Descent Parser Implementation

- For terminals, create function match(a)
  - If lookahead is a it consumes the lookahead by advancing the lookahead to the next token, and returns
  - Otherwise fails with a parse error if lookahead is not a
  - In algorithm descriptions, consider parse_a, parse_term(a) to be aliases for match(a)
- For each nonterminal N, create a function parse_N
  - Called when we're trying to parse a part of the input which corresponds to (or can be derived from) N
  - parse_S for the start symbol S begins the parse

Parser Implementation (cont.)

- The body of parse_N for a nonterminal N does the following
  - Let $N \rightarrow \beta_1 \mid \ldots \mid \beta_k$ be the productions of N
    - Here β_i is the entire right side of a production- a sequence of terminals and nonterminals
  - Pick the production $N \rightarrow \beta_i$ such that the lookahead is in First(β_i)
    - It must be that First(β_i) ∩ First(β_j) = ∅ for $i \neq j$
    - If there is no such production, but $N \rightarrow \varepsilon$ then return
    - Otherwise fail with a parse error
  - Suppose $\beta_i = \alpha_1 \alpha_2 \ldots \alpha_n$. Then call parse_$\alpha_1$(); ... ; parse_$\alpha_n$() to match the expected right-hand side, and return

Parser Implementation (cont.)

- Parse is built on procedure calls
- Procedures may be (mutually) recursive

Recursive Descent Parser

- Given grammar $S \rightarrow xyz \mid abc$
  - First(xyz) = { x }, First(abc) = { a }
- Parser

```
parse_S( ) {
    if (lookahead == "x") {
        match("x"); match("y"); match("z");      // S → xyz
    }
    else if (lookahead == "a") {
        match("a"); match("b"); match("c");      // S → abc
    }
    else error( );
}
```


Recursive Descent Parser

- Given grammar $S \rightarrow A \mid B \quad A \rightarrow x \mid y \quad B \rightarrow z$
  - First(A) = { x, y }, First(B) = { z }
- Parser

```
parse_S( ) {
    if ((lookahead == "x") ||
        (lookahead == "y"))
        parse_A( );    // S → A
    else if (lookahead == "z")
        parse_B( );    // S → B
    else error( );
}

parse_A( ) {
    if (lookahead == "x")
        match("x");    // A → x
    else if (lookahead == "y")
        match("y");    // A → y
    else error( );
}

parse_B( ) {
    if (lookahead == "z")
        match("z");    // B → z
    else error( );
}
```

Example

E → id = n | { L }   First(E) = { id, "{" }

L → E ; L | ε

```
parse_E( ) {
    if (lookahead == "id") {
        match("id");
        match("=");    // E → id = n
        match("n");
    }
    else if (lookahead == "{") {
        match("{");
        parse_L( );    // E → { L }
        match("}");
    }
    else error( );
}
```

```
parse_L( ) {
    if ((lookahead == "id") ||
        (lookahead == "{")) {
        parse_E( );
        match(";"); // L → E ; L
        parse_L( );
    }
    else ;          // L → ε
}
```


Things to Notice

- If you draw the execution trace of the parser
  - You get the parse tree
- Examples
  - Grammar
    - S → xyz
    - S → abc
  - String "xyz"

```
parse_S( )           S
    match("x")      /|\
    match("y")     x y z
    match("z")
```

- Grammar
  - S → A | B
  - A → x | y
  - B → z
- String "x"

```
parse_S( )           S
    parse_A( )       |
        match("x")   A
                     |
                     x
```


Things to Notice (cont.)

- This is a predictive parser
  - Because the lookahead determines exactly which production to use
- This parsing strategy may fail on some grammars
  - Possible infinite recursion
  - Production First sets overlap
  - Production First sets contain ε
- Does not mean grammar is not usable
  - Just means this parsing method not powerful enough
  - May be able to change grammar

Left Factoring

- Consider parsing the grammar E → ab | ac
  - First(ab) = a
  - First(ac) = a
  - Parser cannot choose between RHS based on lookahead!
- Parser fails whenever $A \rightarrow \alpha_1 \mid \alpha_2$ and
  - $First(\alpha_1) \cap First(\alpha_2)$!= ε or ∅
- Solution
  - Rewrite grammar using left factoring

Left Factoring Algorithm

- Given grammar
  - $A \rightarrow x\alpha_1 \mid x\alpha_2 \mid \ldots \mid x\alpha_n \mid \beta$
- Rewrite grammar as
  - $A \rightarrow xL \mid \beta$
  - $L \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$
- Repeat as necessary
- Examples
  - S → ab | ac ⇨ S → aL L → b | c
  - S → abcA | abB | a ⇨ S → aL L → bcA | bB | ε
  - L → bcA | bB | ε ⇨ L → bL' | ε L' → cA | B

Left Recursion

- Consider grammar S → Sa | ε
  - First(Sa) = a, so we're ok as far as which production
  - Try writing parser

```
parse_S( ) {
    if (lookahead == "a") {
        parse_S( );
        match("a");    // S → Sa
    }
    else { }
}
```

  - Body of parse_S( ) has an infinite loop
    - if (lookahead = "a") then parse_S( )
  - Infinite loop occurs in grammar with left recursion

Right Recursion

- Consider grammar $S \rightarrow aS \mid \varepsilon$
  - Again, First(aS) = a
  - Try writing parser

```
parse_S( ) {
    if (lookahead == "a") {
      match("a");
      parse_S( );    // S → aS
    }
    else { }
}
```

  - Will parse_S( ) infinite loop?
    - Invoking match( ) will advance lookahead, eventually stop
  - Top down parsers handles grammar w/ right recursion

Algorithm To Eliminate Left Recursion

- Given grammar
  - $A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots \mid A\alpha_n \mid \beta$
    - Why must β exist?
- Rewrite grammar as
  - $A \rightarrow \beta L$
  - $L \rightarrow \alpha_1 L \mid \alpha_2 L \mid \ldots \mid \alpha_n L \mid \varepsilon$
- Replaces left recursion with right recursion
- Repeat as necessary

Eliminating Left Recursion (cont.)

- Examples
  - $S \rightarrow Sa \mid \varepsilon$ ⇨ $S \rightarrow L$ $L \rightarrow aL \mid \varepsilon$
  - $S \rightarrow Sa \mid Sb \mid c$ ⇨ $S \rightarrow cL$ $L \rightarrow aL \mid bL \mid \varepsilon$
- May need more powerful algorithms to eliminate mutual recursion leading to left recursion
  - $S \rightarrow Aa \mid b$
    $A \rightarrow Sb$

Expr Grammar for Top-Down Parsing

$E \rightarrow T\ E'$
$E' \rightarrow \varepsilon \mid + E$
$T \rightarrow P\ T'$
$T' \rightarrow \varepsilon \mid * T$
$P \rightarrow n \mid (E)$

- Notice we can always decide what production to choose with only one symbol of lookahead

Tradeoffs with Other Approaches

- Recursive descent parsers are easy to write
  - The formal definition is a little clunky, but if you follow the code then it's almost what you might have done if you weren't told about grammars formally
  - They're unable to handle certain kinds of grammars
- Recursive descent is good for a simple parser
  - Though tools can be fast if you're familiar with them
- Can implement top-down predictive parsing as a table-driven parser
  - By maintaining an explicit stack to track progress

Tradeoffs with Other Approaches

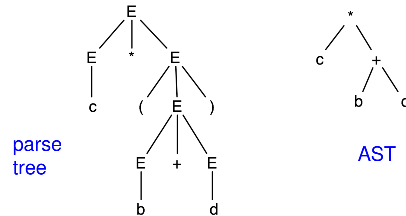
- More powerful techniques need tool support
  - Can take time to learn tools (lex/flex, yacc/bison)
- Main alternative is bottom-up, shift-reduce parser
  - Replaces RHS of production with LHS (nonterminal)
  - Example grammar
    - $S \rightarrow aA$, $A \rightarrow Bc$, $B \rightarrow b$
  - Example parse
    - $abc \Rightarrow aBc \Rightarrow aA \Rightarrow S$
    - Derivation happens in reverse
  - Something to look forward to in CMSC 430

What's Wrong With Parse Trees?

- Parse trees contain too much information
  - Example
    - Parentheses
    - Extra nonterminals for precedence
  - This extra stuff is needed for parsing
- But when we want to reason about languages
  - Extra information gets in the way (too much detail)

Abstract Syntax Trees (ASTs)

- An abstract syntax tree is a more compact, abstract representation of a parse tree, with only the essential parts

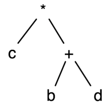


parse tree

AST

Abstract Syntax Trees (cont.)

- Intuitively, ASTs correspond to the data structure you'd use to represent strings in the language
  - Note that grammars describe trees
  - So do OCaml datatypes (which we'll see later)
  - E → a | b | c | E+E | E-E | E*E | (E)

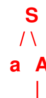


Producing an AST

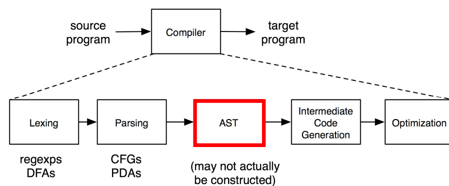
- To produce an AST, we can modify the parse() functions to construct the AST along the way
  - match(a) returns an AST node (leaf) for a
  - Parse_A returns an AST node for A
    - AST nodes for RHS of production become children of LHS node
- Example
  - S → aA

```
Node parse_S( ) {
    Node n1, n2;
    if (lookahead == "a") {
      n1 = match("a");
      n2 = parse_A( );
      return new Node(n1,n2);
    }
}
```

S
/ \
a A



The Compilation Process



Summary

- Learned a little about parsing
  - Recursive descent parser
  - Predictive parsing using FIRST sets
- Rewriting grammars for predicative parsing
  - Left factoring
  - Eliminating left recursion
- Abstract syntax trees (ASTs)