

CMSC330 Fall 2009 Example Quiz #3

Name _____

Discussion Time (circle one): 10am 11am 12pm 1pm 2pm 3pm

Instructions

- You have 25 minutes for this quiz.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

1. (9 pts) Lambda calculus

(3 pts) Find all free (unbound) variables in the following λ -expression

a. $(\lambda a. c b) \lambda b. a$

(3 pts each) Evaluate the following λ -expressions as much as possible

b. $(\lambda x. \lambda y. y x) a b$

c. $(\lambda z. z x) (\lambda y. y x)$

2. (16 pts) Lambda calculus encodings

Prove the following using the appropriate λ -calculus encodings, given:

$$1 = \lambda f. \lambda y. f y$$

$$2 = \lambda f. \lambda y. f (f y)$$

$$3 = \lambda f. \lambda y. f (f (f y))$$

$$4 = \lambda f. \lambda y. f (f (f (f y)))$$

$$M * N = \lambda x. (M (N x))$$

$$Y = \lambda f. (\lambda x. f (x x)) (\lambda x. f (x x))$$

$$\text{succ} = \lambda z. \lambda f. \lambda y. f (z f y)$$

a. (10 pts) Show that $2 * 2 \Rightarrow^* 4$ // $2 * 2$ reduces to 4 using encodings

b. (6 pts) Show that $(Y \text{succ}) x \Rightarrow^* \text{succ} (Y \text{succ}) x$ // using encoding for Y
// you do not need to expand succ
// you may assume $((\lambda x. \text{succ} (x x)) (\lambda x. \text{succ} (x x))) \Rightarrow (Y \text{succ})$