CMSC330 Fall 2009 Quiz #3 Solution

Name _____

Discussion Time (circle one): **10am 11am 12pm 1pm 2pm 3pm**

Instructions

- You have 20 minutes for this quiz.
- For partial credit, show all of your work and clearly indicate your answers.
- Write neatly. Credit cannot be given for illegible answers.

Circle all free (unbound) variables in the following λ-expression

1. (2 pts) ((λa. **b** λb. a b **c**) **b** **a**)

Evaluate the following λ-expressions as much as possible. Show each beta-reduction.

2. (2 pts) (λx.λy.x x) a b **=> (λy.a a) b => a a**

3. (3 pts) (λx.λy.y x y) y **=> λz.z y z** **(or any char for z except y)**

4. (3 pts) (λx.x a) (λy. λz. b z y) c **=> ((λy. λz. b z y) a) c => (λz. b z a) c => b c a**

5. (10 pts) Using encodings, show iszero 2 =>* false. Show each beta-reduction.
   You do not need to expand true & false.
   =>* indicates 0 or more steps of beta-reduction.

iszero 2 => (λg.g (λy.false) true) 2 **// replace g with 2**
=> 2 (λy.false) true **// apply encoding for 2**
=> (λf.λy.f (f y)) (λy.false) true **// alpha-rename y to z**
=> (λf.λz.f (f z)) (λy.false) true **// replace f with λy.false**
=> (λz. (λy.false) ((λy.false) z)) true **// replace z with true**
=> (λy.false) ((λy.false) true) **// replace y with (λy.false) true**
=> false

true = λx. λy.x
false = λx. λy.y
if a then b else c = a b c
not = λx.((x false) true)
and = λx. λy.((xy) false)
or = λx. λy.((x true) y)
iszero = λg.g (λy.false) true
succ = λz.λf.λy.f (z f y)
1 = λf.λy.f y
2 = λf.λy.f (f y)
3 = λf.λy.f (f (f y))
4 = λf.λy.f (f (f (f y)))