Fluid Simulation Overview

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Outline

- Introduction
- Fluid Characteristics
- Navier-Stokes Equation
- Eulerain vs. Lagragian approach
- Fluid Simulation Stages
- Solid Fluid Coupling
- Read-time fluids

Introduction

- Application
  - Computer Games
  - Scientific Computation
  - Special Movie Effects
  - **Medical Simulation (Blood Flow)**
- What can we achieved so far?
  - Smoke
  - Granular flow (sand)
  - Newtonian fluid (Water, ocean)
  - **Non-Newtonian fluid (Blood, Honey)**
  - Microscopic phenomena

Characteristics - Basic Fluid Properties

- Vector Field
  - Velocity (***v***)
  - Pressure (***p***)
  - Additional Force (***f***)
  - Surface Tension (***t***)
- Scalar Field
  - Density (ρ)
  - Viscosity (ν)

Characteristics – Fluid Types

- Incompressible vs. Compressible Fluids
  - Incompressible Fluids: Fluids doesn't change volume very much
  - Compressible Fluids: Fluids change their volume significantly
- Viscous vs. Inviscid Fluids(ideal):
  - Viscous Fluids: Fluids tend to resist a certain degree of deformation
  - Inviscid Fluids: Fluids don't have resistance to the shear stress
- Turbulent vs. Laminar (streamline) Flow:
  - Turbulent Flow: Flow that appears to have chaotic and random changes
  - Laminar Flow: Flow that has smooth behavior
- Newtonian vs. Non-Newtonian Fluids:
  - Fluids continue to flow, regardless of the force acting on it
  - Fluids that have non-constant viscosity

Naiver-Stokes Equation

- Momentum Equation

$$\mathbf{u}_t = k\nabla^2\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \mathbf{f}$$

Change in velocity | Diffusion/Viscosity | Advection | Pressure | Body Forces

- Incompressibility

$$\nabla\cdot\mathbf{u}=0$$

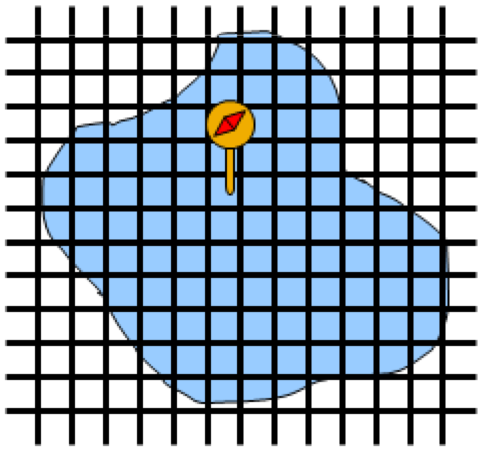
u: the velocity field

k: kinematic viscosity

- Common techniques for solving Navier Stoke's equation:
  - **Eulerian approach (grid-based)**
  - **Lagrangian approach (particle-based)**
  - Spectral method
  - Lattice Boltzmann method

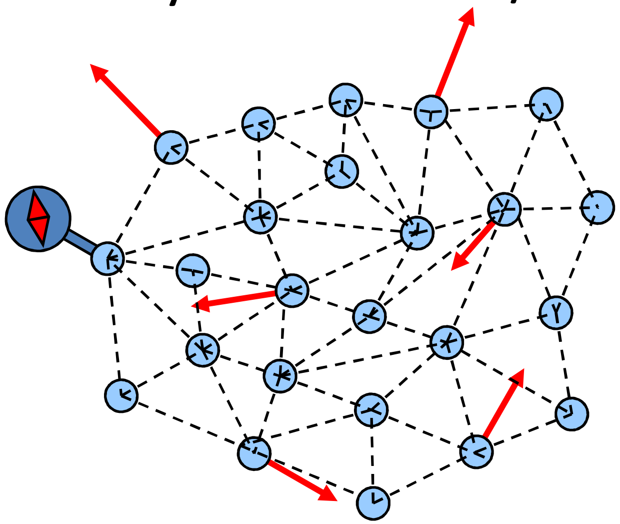
Eulerian Approach

- Discretize the domain using finite differences
- Define scalar & vector fields on the grid
- Use the operator splitting technique to solve each term separately
- Evaluation:
  - ✓ Derivative approximation
  - ✓ Adaptive time step/solver
  - ✗ Memory usage & speed
  - ✗ Grid artifact/resolution limitation

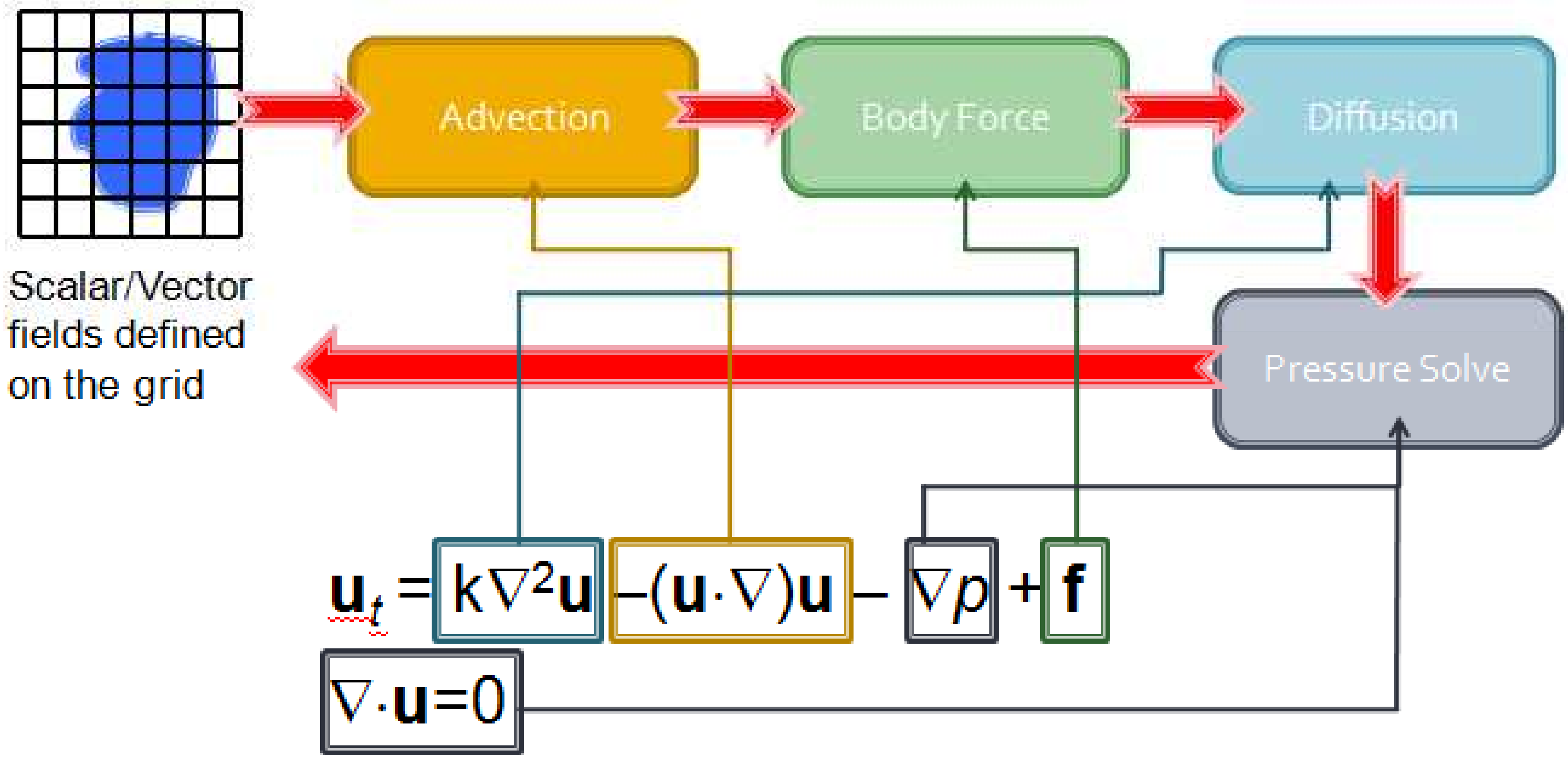


Lagrangian Approach

- Treat the fluid as discrete particles
- Apply interaction forces (i.e. pressure/viscosity) according to certain pre-defined smoothing kernels
- Evaluations:
  - ✓ Mass / Momentum conservation
  - ✓ More intuitive
  - ✓ Fast, no linear system solving
  - ✗ Connectivity information/Surface reconstruction



Fluid Simulation Stages – Main Loop



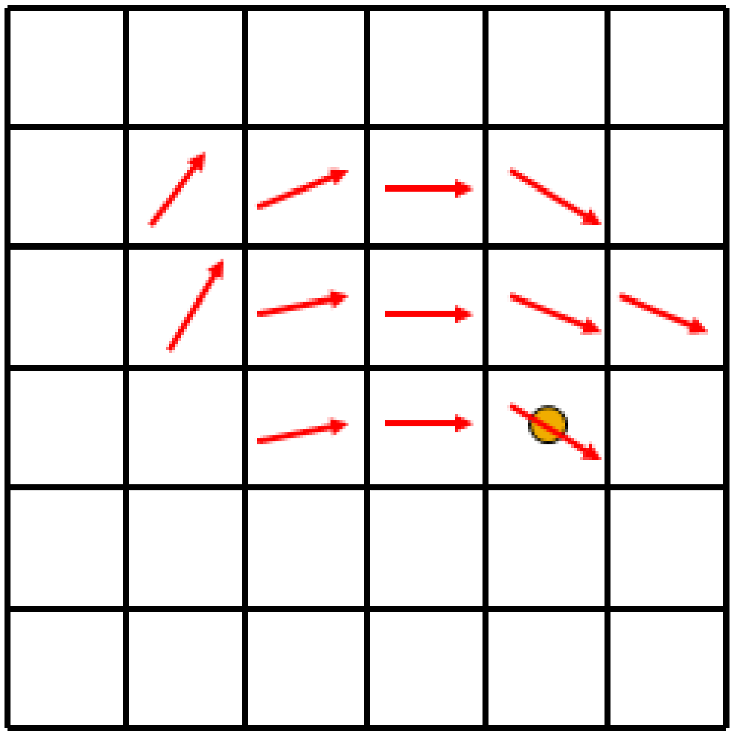
Fluid Simulation Stages – Advection I

- Sometimes called "Convection" or "Transport"
- Define how a quantity moves with the underlying velocity field
- This term ensures the conservation of momentum
- Advection equation:

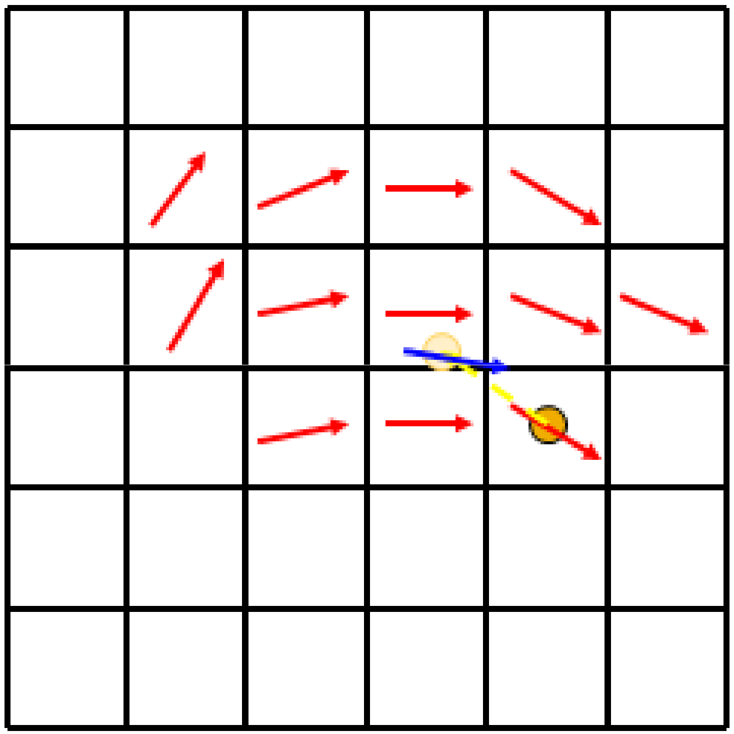
$$u_t = -(u \cdot \nabla u)$$

- Approaches:
  - Forward Euler (unstable)
  - Semi-Lagragian advection (stable for large time steps, but suffers from the dissipation issue)

Fluid Simulation Stages – Advection II



Forward Euler Advection



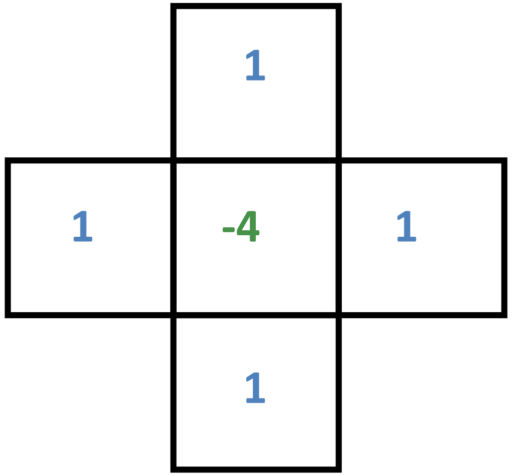
Semi-Lagragian Advection

Fluid Simulation Stages – Diffusion I

- Define how a quantity in a cell inter-changes with its neighbors
- Diffusion = Blurring
- The viscous fluid can be achieved by applying diffusion to the velocity field

Figures from [Carlson, Mucha, Turk] Melting and Flowing, SCA 02

Fluid Simulation Stages – Diffusion II



- Diffusion equation:

$$u_t = k \cdot \nabla^2 u$$

- Approaches:
  - Explicit formulation

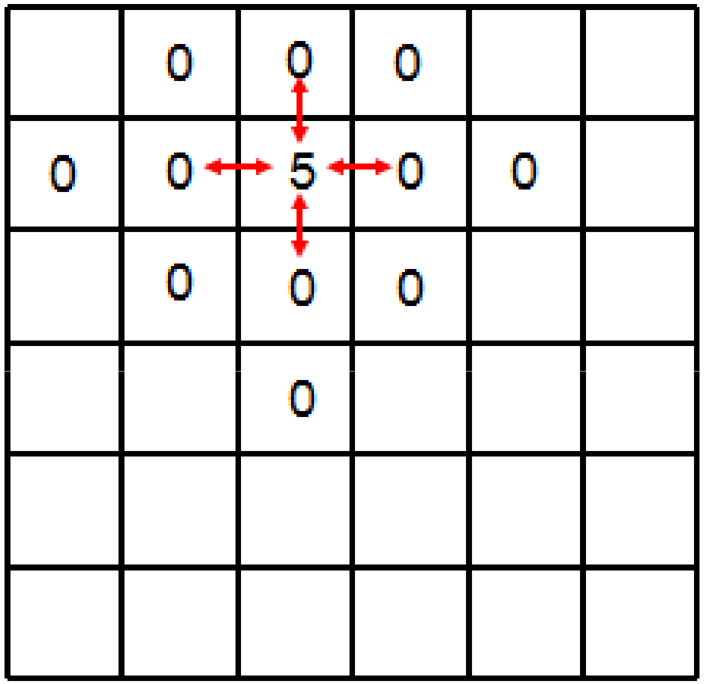
$$u_t = k \cdot \Delta t \cdot (u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4 \cdot u_{i,j})$$

  - Implicit formulation (for high viscosity)

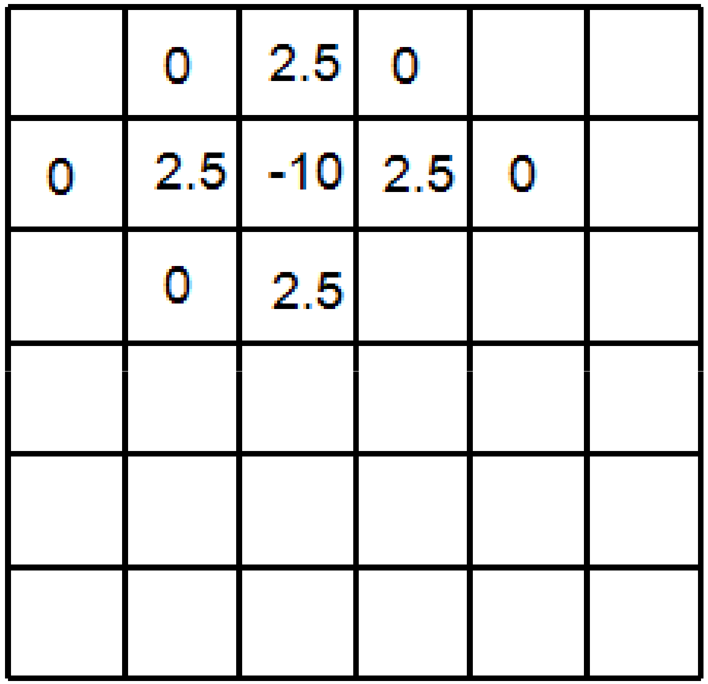
$$u^n{}_{i,j} = u^{n+1}{}_{i,j} - k \cdot \Delta t \cdot (u^{n+1}{}_{i+1,j} + u^{n+1}{}_{i-1,j} + u^{n+1}{}_{i,j+1} + u^{n+1}{}_{i,j-1} - 4 \cdot u^{n+1}{}_{i,j})$$

Unknowns

Fluid Simulation Stages – Diffusion III



Before the diffusion

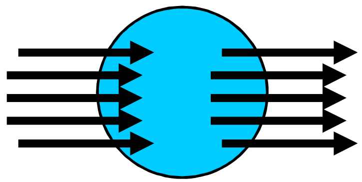


After the diffusion (k = 0.5, time step size =1)

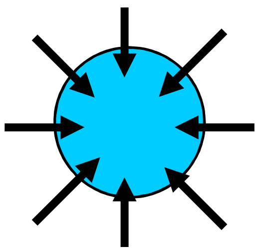
Fluid Simulation Stages – Pressure Solver I

- It’s sometimes called “Pressure Projection”
- What does the pressure do?
  - Keep the fluid at constant volume (incompressible, conservation of mass).
  - Make sure the velocity field stays divergence-free

$$\sum_{all\_faces} flux = 0$$



Incompressible



Compressible

Fluid Simulation Stages – Pressure Solver II

- Equation to solve:

$$u^{n+1} = u^n - \frac{1}{\rho}\nabla p \qquad \text{s.t.} \quad \nabla \cdot u^{n+1} = 0 \quad \cdots (1)$$

Unknowns

- How to solve for pressure:
  - Taking divergence of both sides of (1), we will have

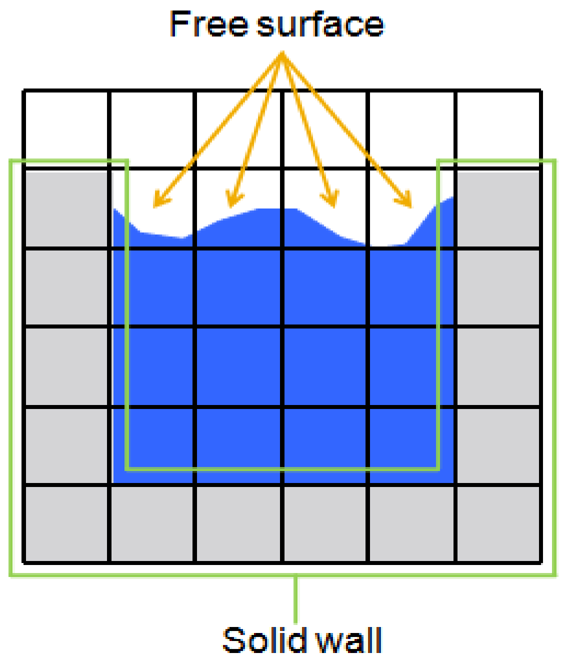
    $$\frac{1}{\rho}\nabla^2 p = \nabla \cdot u^n \quad \text{(Poisson Equation)}$$

  - Build a system of equations and solve Ap = d using an iterative method such as Conjugate Gradient
  - Update the velocity field from the pressure gradient

Fluid Simulation Stages – Pressure Solver III

- What about the pressure on boundary nodes?
  - Free surface: The fluid can evolve freely (p = 0)
  - Solid wall: The fluid can't penetrate the wall but can flow freely in tangential directions (Neumann BC)

$$u_{boundary} \cdot n = u_{solid} \cdot n$$

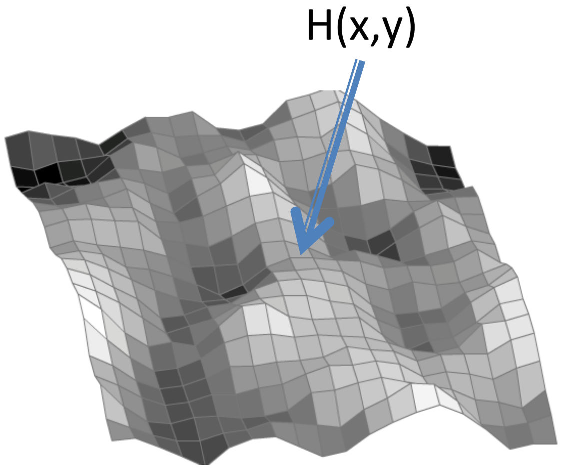


Solid Fluid Coupling

- One-way coupling:
  - Solid-Fluid interaction: The fluid has no influence on the solid
  - Fluid-Solid interaction: The solid has no influence on the fluid
- Two-way coupling:
  - Manipulate the boundary conditions
  - Finite Element techniques: ALE & DLM
  - Rigid Fluid: Treat the solid as fluids and enforce the rigidity constraint

Real-time Fluids

- Principles:
  - Cheap to compute
  - Low memory consumption
  - Stability
  - Plausibility
  - Interactivity



- Common techniques:
  - Procedural water: Superimpose sine waves of a variety of amplitudes and directions.
  - Height field approximations: If the surface is the only interest, it can be represented using a 2D height field and animated by 2d wave equations with interaction forces
  - Particle systems: This approach is good at simulating a small amount of water such as a puddle, a bubble or splashing fluids

$$f(x_i, x_j) = \left(\frac{k_1}{|x_i - x_j|^m} - \frac{k_2}{|x_i - x_j|^n} \right) \cdot \frac{x_i - x_j}{|x_i - x_j|}$$

Agenda

- Sep 15th - Particle-Based Fluid Simulation for Interactive Applications
- Sep 15th - Hardware-Aware Analysis and Optimization of Stable Fluids
- Sep 22nd – Direct Forcing for Lagrangian Rigid-Fluid Coupling
- Sep 22nd – Directable, High-Resolution Simulation of Fire on the GPU