Direct Forcing for Lagrangian Rigid-Fluid Coupling

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Introduction

- The interaction of fluids with fixed and moving obstacles
- Particle based method: Smoothed Particle Hydrodynamics (SPH)
  - Preserve the local nature given in many Lagrangian fluid simulations
  - Usefulness of Lagrangian fluids for irregular domains
- Direct Forcing: Control Forces are incorporated in the discretized momentum equations in order to obtain specific relative velocities at a boundary in each time step
- Local nature of the employed SPH method is preserved by the boundary handling
- Extend previous Lagrangian boundary approaches
Highlight

• propose a novel boundary handling algorithm for particle-based fluids
• Based on a predictor-corrector scheme for both velocity and position, one- and two-way coupling with rigid bodies can be realized.
• Different slip conditions can be realized and non-penetration is enforced.
• Direct forcing is employed to meet the desired boundary conditions and to ensure valid states after each simulation step
Fluid Model

• A Corrected SPH formulation (CSPH)
  – Original Version: 
    \[ \langle f(x) \rangle = \sum_b \frac{m_b}{\rho_b} f(x_b) W_b(x). \]
  – Avoids inaccurate pressures at boundaries
  – Using the constant correction technique for SPH

• Use the reformulated Euler equation to compute the fluid dynamics

  \[ \frac{dv_a}{dt} = -\sum_b m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_x W_b(x_a) + g \]

  \[ \Pi_{ab} = -\nu \left( \frac{v_{ab}^T x_{ab}}{|x_{ab}|^2 + \delta h^2} \right). \]

  the viscosity term \quad g: External forces

• The weakly compressible pressure formulation employing the Tait equation

  \[ P = \frac{\rho_0 c_s^2}{7} \left( \left( \frac{\rho}{\rho_0} \right)^7 - 1 \right). \]

  \[ \rho: \text{ The current density} \]
  \[ \rho_0: \text{ The initial density} \]
  \[ c_s: \text{ The speed of sound} \]
Rigid Body Model

• A particle representation for arbitrary rigid-body surfaces
• Generated in a preprocessing step using a distance field
• Boundary handling approach could be combined with alternative representations such as triangle meshes or distance fields

Figure. Triangulated surface and particle representation of the teddy model.
Boundary Handling Overview

• A novel technique to enforce boundary conditions for particle-based rigid-fluid contacts
• Allows the control of both the normal and tangential relative velocities and the relative positions effectively to realize various boundary conditions.
• The relative velocities and positions are controlled in separate sub-steps
• Enforcing the desired velocities and positions is realized using a direct forcing approach
• Implementing the direct forcing in a predictor-corrector fashion to address non-penetration problem.
• Advance the positions without boundary forces and perform the collision detection on that advanced positions.
Controlling the relative velocity

- A contact point
  \[ x_{cp} = x_p + r_p n \]
- The rigid-body velocity at the contact point
  \[ v_{cp}'(t+h) = v_c'(t+h) + \omega_c'(t+h) \times r_c'(t+h) \]
  with \( r_c'(t+h) = x_{cp}'(t+h) - x_c'(t+h) \) being the relative position of \( x_{cp}'(t+h) \) with respect to the center of mass \( x_c'(t+h) \) of the rigid body. \( v_c'(t+h) \) and \( \omega_c'(t+h) \) denote the linear and angular velocities of the rigid body, respectively.
- Impose a boundary condition on the relative velocity
  \[ v_r(t+h) := v_F(t+h) - v_{cp}'(t+h) = \varepsilon [v_r'(t+h)]_t - \delta [v_r(t)]_n \]
  \( v_r = v_F - v_{cp} \)
  The first term of controls the slip. It can be used to damp the relative tangential velocity of the fluid and the rigid body. The second term of controls the elasticity of the collision.
- \( \delta = 1.0 \) perfectly elastic collision \( \delta = 0.0 \) perfectly inelastic collision
- Current normal velocity
  \[ [v_r(t)]_n = (v_r(t) \cdot n)n \]
- Uncontrolled tangential velocity
  \[ [v_{\tau r}'(t+h)]_t = v_{\tau r}'(t+h) - [v_{\tau r}'(t+h)]_n \]
- Use the predicted velocity of the subsequent time step to properly consider accelerations due to body forces such as gravity.
Velocity Update

• **For a simple Euler step:**
  – The constrained velocities for the fluid particle
    \[ \mathbf{v}_i(t + h) = \mathbf{v}_i^*(t + h) + \frac{h}{m_i} \mathbf{F}_i \]
  – The constrained velocities for the rigid body at the contact point
    \[ \mathbf{v}_{c,i,j}(t + h) = \mathbf{v}_{c,i,j}^*(t + h) - \frac{h}{m_c} \mathbf{F}_i + h \dot{\mathbf{r}}_i^*(t + h) \mathbf{I}^{-1}(t) \dot{\mathbf{r}}_i^*(t + h) \mathbf{F}_i \]

• **Local approach:** If we have $k$ fluid particles in contact with a single rigid body, solve each contact separately using
  \[
  \mathbf{F}_i = \frac{1}{h} \left[ \left( \frac{1}{m_i} + \frac{1}{m_c} \right) \mathbf{E}_3 + \dot{\mathbf{r}}_i^T(t + h) \mathbf{I}^{-1}(t) \dot{\mathbf{r}}_i^*(t + h) \right]^{-1} \dot{\mathbf{v}}_i, \quad \dot{\mathbf{v}}_i := \epsilon [\mathbf{v}_{c,i,j}(t + h)]_t - \delta [\mathbf{v}_{c,i,j}(t)]_n - \mathbf{v}_{c,i,j}^*(t + h)
  \]
  and simply add up the forces and torques on the rigid body, we term this local approach.

• **Global approach:** Now, we assume that a single rigid body is in contact with $k$ fluid particles and we want to enforce all contact velocities simultaneously. This is termed global approach.
  \[
  \mathbf{F} = - \sum_i \mathbf{F}_i, \quad \mathbf{\tau} = - \sum_i \mathbf{r}_i^*(t + h) \times \mathbf{F}_i
  \]
  \[
  \begin{bmatrix}
  \frac{1}{m_c} \mathbf{F} \\
  \mathbf{I}^{-1}(t) \mathbf{\tau}
  \end{bmatrix} = 
  \begin{bmatrix}
  - \sum \frac{m_i}{h} \dot{\mathbf{v}}_i \\
  - \sum \frac{m_i}{h} \dot{\mathbf{r}}_i^*(t + h) \dot{\mathbf{v}}_i
  \end{bmatrix}
  \]
  \[
  \begin{bmatrix}
  (m_c + \sum m_i) \mathbf{E}_3 \\
  \sum m_i \dot{\mathbf{r}}_i^T(t + h) \mathbf{I}(t) + \sum m_i \dot{\mathbf{r}}_i^T(t + h) \dot{\mathbf{r}}_i^T(t + h)
  \end{bmatrix}
  \]
  \[
  \mathbf{A} := 
  \begin{bmatrix}
  \sum m_i \dot{\mathbf{r}}_i^T(t + h) \mathbf{I}(t) + \sum m_i \dot{\mathbf{r}}_i^T(t + h) \dot{\mathbf{r}}_i^T(t + h)
  \end{bmatrix}
  \]
Position Update

- Enforce non-penetration of the fluid particles with respect to the boundary by controlling the position of the fluid particles at the boundary in a separate sub-step.

- Enforce the centers of the boundary fluid particles with radius $r_i$ to retain a distance $r_i^*$ to the contact point $x_{cp,i}$.

\[ x_{i}(t+h) + h j_i - x_{cp,i}(t+h) \cdot n = r_F. \]

- The control impulses

\[ j_i = \frac{1}{h} \left( (x_{cp,i}(t+h) - x_i^{**}(t+h)) \cdot n + r_F \right) n. \]

- The position update leads to a higher compression of the fluid at the boundary layer.

- Higher density ratios are rapidly balanced in subsequent time steps with the employed weakly compressible pressure approach.
Two-way coupling

• A predictive integration step for the fluid and the rigid bodies
• In the correction the position update step, we only consider fluid particles and rigid bodies that are in contact.
• Three collision detection steps are performed.

Pseudocode for two-way coupled moving boundaries

Require: n fluid particles, m rigid bodies
1: Detect fluid-fluid collisions
2: Calculate fluid and rigid-body forces
3: Integrate fluid and rigid body (prediction) 
   \( x(t) \rightarrow x^*(t+h), v(t) \rightarrow v^*(t+h) \)
4: Detect rigid-fluid collisions
5: Calculate net force \( F \) and net torque \( \tau \)
6: Integrate fluid and rigid body (correction) 
   \( x^*(t) \rightarrow x^{**}(t+h), v^*(t+h) \rightarrow v(t+h) \)
7: if (any contacts in 4) then
8: Detect rigid-fluid collisions
9: Correct fluid positions \( x^{**}(t+h) \rightarrow x(t+h) \)
10: end if
One-way coupling and Static Boundary

• One-way coupling: the solid influences the fluid, but not vice versa

• Applicable Cases
  – The influence of the fluid on the solid is small and could be neglected
  – The solid does not move at all

• The rigid-body velocity at the contact point simplifies to:
  – One-way coupling $v_{cp,i}(t+h) = v_{cp,i}(t+h)$
  – Static boundaries $v_{cp,i}(t+h) = v_{cp,i}(t) = 0$

• Fluid velocity can be calculated from the boundary conditions in one step, no need to solve a system of equations
• Save one collision detection step compared to the two-way coupling
Implementation

• The time step for the simulation is chosen according to the Courant-Friedrichs-Lewy (CFL) convergence condition
• Need to ensure that two particles do not move more than their diameter toward each other in one time step.
• Two free parameters: the damping parameter $\epsilon$ and $\delta$, always in the interval $[0, 1]$.
• Efficient detection of particle-particle contacts:
  – Spatial subdivision
  – Store the results in a hash table only
• Update the information of particles if their grid cell has changed.
  – Temporal coherence significantly speeds up the insertion of particles into the hash table.
Results

• Make use of both the local and global approaches for updating the boundary velocity
• Intel Dual Core 2.13 GHz with 4 Gbytes of RAM, running a single-threaded version of the simulation.
• Experiments (**Videos**)
  – Comparison with the penalty approach: Sinking Ship
    • Penalty methods offer only limited control.
    • Penalty methods can only react in a subsequent time step, if a penetration has already occurred
  – Accuracy: simulate a vessel falling into a large basin of fluid
  – Slip Condition
    • Free Slip
    • Partial Slip
    • Full Slip
  – One-Way Solid-to-Fluid Coupling: High-velocity impact of an asteroid
  – Buoyancy Effects: Cuboids of different densities dropped into a fluid.
  – Drag Effects: High viscosity vs. Low viscosity
  – Two-Way Coupling:
    • Sinking Ship (Local approach)
    • Skipping Stone (Global approach)
    • complex-shaped rigid objects are floating on a wave (Local approach)
Conclusion & Future Work

• An efficient Lagrangian method for the handling of fixed and moving boundaries
• Direct forcing is employed to realize a large range of slip and Neumann boundary conditions.
• Can be used for one- and two-way coupling with arbitrarily shaped boundaries that are represented with particles.
• Static and dynamic forces are properly taken into account to allow for buoyancy and drag effects.
• Overlaps of fluid and rigid-body particles are avoided.
• Offers a greater amount of control, ensures non-penetration, and does not introduce stiffness to the system
• It is computationally efficient and scales linearly in the number of contact points
• Currently do not handle simultaneous contact of a single fluid particle with more than one rigid body and simultaneous contact of several rigid bodies in a fluid.
Thank you for your attention.😊
Questions???