Folding meshes: Hierarchical mesh segmentation based on planar symmetry

A Concise and Provably Informative Multi-Scale Signature Based on Heat Diffusion

André Maximo

November, 2009
CMSC 828V
motivation

definition

similar

1. Related in appearance or nature; alike though not identical.
2. (Mathematics) Having corresponding angles equal and corresponding line segments proportional.

symmetry

1. Exact correspondence of form and constituent configuration on opposite sides of a dividing line or plane or about a center or an axis.
2. A relationship of characteristic correspondence, equivalence, or identity among constituents of an entity or between different entities.
motivation

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motivation

main applications

Segmentation
Compression
Scan completion
Shape analysis
Mesh Editing
Shape Matching
...

motivation

cutting meshes
hks
folding meshes

Eurographics Symposium on Geometry Processing (2006)

Folding meshes: Hierarchical mesh segmentation based on planar symmetry

Patricio Simari, Evangelos Kalogerakis, Karan Singh

DGP Lab, Department of Computer Science, University of Toronto
folding meshes

overview

1- Symmetric region detection
folding meshes

overview

2- Planes of symmetry
folding meshes

overview

3- Folding Tree
folding meshes

symmetric region detection

“A symmetric surface’s planes of symmetry are orthogonal to the eigenvectors of its covariance matrix and contain its center of mass.”
folding meshes

symmetric region detection

“A symmetric surface’s planes of symmetry are orthogonal to the eigenvectors of its covariance matrix and contain its center of mass.”

Consider a candidate symmetry plane $p$ and let $d_i$ be the distance of vertex $v_i$ to the reflected mesh with respect to $p$. 
folding meshes

symmetric region detection

“A symmetric surface’s planes of symmetry are orthogonal to the eigenvectors of its covariance matrix and contain its center of mass.”

Consider a candidate symmetry plane $p$ and let $d_i$ be the distance of vertex $v_i$ to the reflected mesh with respect to $p$.

Each $v_i$ is associated a weight $w_i$ according to:

$$
\rho_i = \frac{d_i^2}{\sigma^2 + d_i^2}
\quad w_i = \frac{1}{d_i} \frac{\partial \rho_i}{\partial d_i} = \frac{2\sigma^2}{(\sigma^2 + d_i^2)^2}
$$
“*A symmetric surface’s planes of symmetry are orthogonal to the eigenvectors of its covariance matrix and contain its center of mass.*”

Consider a *candidate symmetry plane* \( p \) and let \( d_i \) be the *distance* of vertex \( v_i \) to the reflected mesh with respect to \( p \).

Each \( v_i \) is associated a *weight* \( w_i \) according to:

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folding meshes

symmetric region detection

“A symmetric surface’s planes of symmetry are orthogonal to the eigenvectors of its covariance matrix and contain its center of mass.”

The plane of symmetry is estimated by the center of mass $m$ and the eigenvectors of the weighted covariance matrix $C$ defined as:

$$m = \frac{1}{s} \sum_{i=1}^{n} w_i v_i$$
$$C = \frac{1}{s} \sum_{i=1}^{n} w_i(v_i - m)(v_i - m)^T$$

where $s = \sum_i w_i$. 
folding meshes

symmetric region detection

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\]

where \( s = \sum_i w_i \).

These eigenvectors and center of mass determine three planes. One with smallest sum cost is chosen.
folding meshes

plane of symmetry

convergence
folding meshes

plane of symmetry

convergence
folding meshes

plane of symmetry

convergence
folding meshes

plane of symmetry

convergence
folding meshes

folding tree

- Encodes the non-redundant regions as well as the reflection planes.

- Created by recursive application of the detection method.

- It can then be unfolded to recover the original shape.
folding meshes

folding tree

(a) (b) (c) (d)
folding meshes

results (planes of symmetry)
folding meshes

results (reconstructing the mesh)
folding meshes

conclusions

- Estimative approach to **find** global and local symmetries.

- **Compact** representation of meshes using the detected symmetries.

drawbacks

- **Translation and rotational** symmetries are ignored.

- **Bad triangles** close to the planes of symmetry on reconstructed meshes.
A Concise and Provably Informative Multi-Scale Signature Based on Heat Diffusion

Jian Sun, Maks Ovsjanikov, Leonidas Guibas

Stanford University
**signature**

1. The name of a person or a mark or sign representing his name, marked by himself or by an authorized deputy.

2. A **distinctive mark**, characteristic, or sound indicating **identity**.
signature

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vertex signature

It is a way to identify a vertex of a mesh.
hks

definition

signature

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vertex signature

It is a way to identify a vertex of a mesh.

similarity

Compare vertex signatures.
vertex signature

- Shape representation uses \((x, y, z)\) coordinates.
vertex signature

- Shape representation uses \((x, y, z)\) coordinates.
- Good for rendering and visualization.
vertex signature

• Shape representation uses \((x, y, z)\) coordinates.

• Good for rendering and visualization.

• Bad for shape analysis:
  – shape matching
  – structure discover
  – similarity
hks

goal

motivation
folding meshes

hks
hks

goal

- concise
- commensurable
- stable
- multi-scale
- isometric invariant
- informative
For any point $x$, the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.
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How heat diffuses over time?
“For any point \( x \), the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.”

How heat diffuses over time?

Heat transferred from \( y \) to \( x \) in time \( t \)

\[
H^t f(x) = \int_M k_t(x, y) f(y) dy
\]
“For any point \( x \), the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.”

Heat Kernel

- recovers geodesic distances: \( d_M^2(x, y) = -4 \lim_{t \to 0} t \log k_t(x, y) \)
- isometric: \( k_t(x, y) = k_t(T(x), T(y)) \) (if applying an isometric transformation \( T \))
“For any point $x$, the **HKS** captures all the information contained in the heat kernel, and characterizes the shape up to isometry.”
For any point $x$, the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.

However, $\{k_t(x, \cdot)\}_t$’s complexity is extremely high and difficult to compare $\{k_t(x, \cdot)\}_t$ with $\{k_t(x', \cdot)\}_t$. 
For any point $x$, the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.

However, $\{k_t(x, \cdot)\}_t$’s complexity is extremely high and difficult to compare $\{k_t(x, \cdot)\}_t$ with $\{k_t(x', \cdot)\}_t$.

**Informative Theorem**

HKS is the restriction of $\{k_t(x, \cdot)\}_t$ to the temporal domain $HKS_x : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ by $HKS_x(t) = k_t(x, x)$.
main contribution

Heat Kernel Signature

“For any point $x$, the HKS captures all the information contained in the heat kernel, and characterizes the shape up to isometry.”

however, $\{k_t(x, \cdot)\}_t$’s complexity is extremely high
difficult to compare $\{k_t(x, \cdot)\}_t$ with $\{k_t(x', \cdot)\}_t$

Informative Theorem

HKS is the restriction of $\{k_t(x, \cdot)\}_t$ to the temporal domain

$HKS_x : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ by $HKS_x(t) = k_t(x, x)$

$$HKS(x) = k_t(x, x) = k_t(x, \cdot)$$
hks

intuitively
hks intuitively
hks

heat kernel function for a small fixed $t$
different time ranges $[t_1, t_2]$
results

motivation folding meshes

hks
conclusions

- Novel **shape representation**: HKS.

- The signature is **multi-scale** and **isometric invariant**.

drawbacks

- It depends on meaningful **parameters** (neighborhood size).

- It is **computationally expensive** (eigen analysis of the mesh).
thank you

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The material content presented is from the respective authors papers and talks, and it was downloaded from the first author's web pages:


**Patricio Simari**: [http://www.cs.jhu.edu/~psimari/](http://www.cs.jhu.edu/~psimari/)