CMSC 131
Object-Oriented Programming I

Sorting/Algorithm Analysis

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This material is based on material provided by Ben Bederson, Bonnie Dorr, Fawzi Emad, David Mount, Jan Plane

Overview

- Sorting/Algorithm Analysis

Sorting and Searching

- **Sorting:** Given a **list** of items from any ordered domain (integers, doubles, Strings, Dates, …) **permute** (rearrange) the elements so they are in **increasing order**
- **Searching**: Given a **list** of items and given a designated **query item** q, determine **whether** q appears in the list, and if so, then **where**
- **Sorting and Searching**: are two of the most fundamental tasks in all of computer science
  - Although these may seem pretty simple, there are many different ways to solve these problems
  - These approaches are quite different in terms of **complexity** and **efficiency**
  - There is a 722 page book (by D. E. Knuth) devoted to just these two topics alone!
- Note: We will talk about Sorting only (one type of Sort: Selection Sort). You will see Search in CMSC 132

Selection Sort

- **Selection Sort**: one of the simplest sorting algorithms known. (**Recall**: an **algorithm** is a method of solving a problem.)
- **Input Argument**: Let **a** denote the array to be sorted. We assume only that the elements of **a** are from any class that implements the **Comparable interface**, that is, it defines a public method **compareTo**

Selection Sort Algorithm

- **Selection Sort**: Works as follows. Let a[0 … n-1] be the array to be sorted.

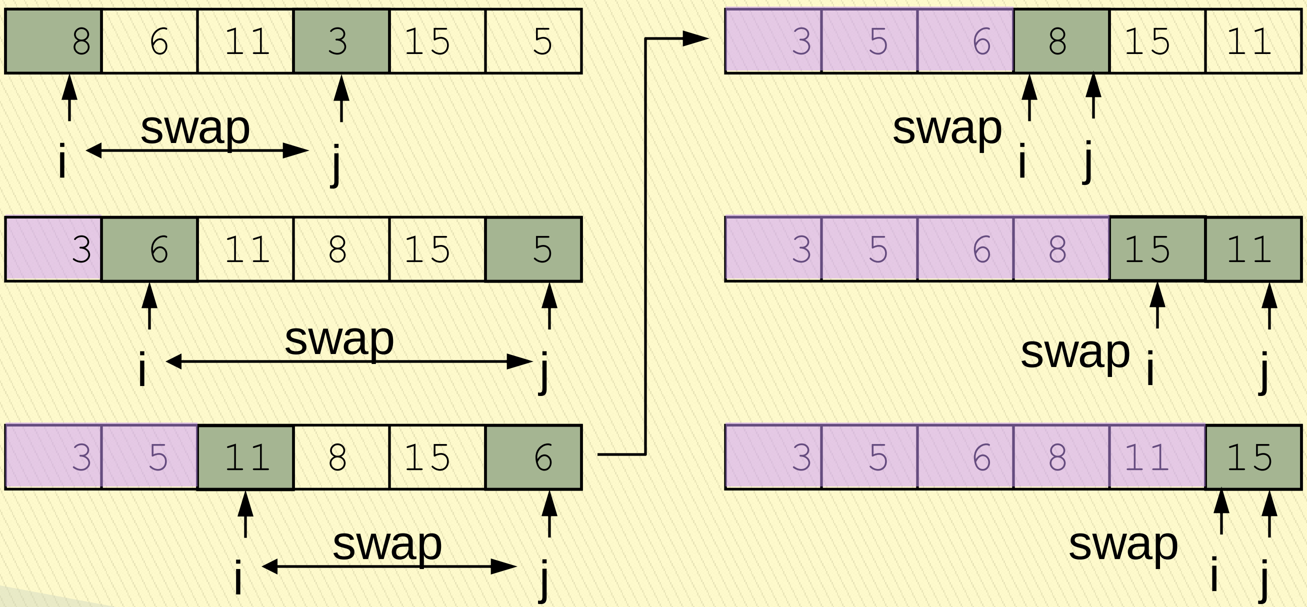
  for (i running from 0 up to n-1) {

  Let j = index of the smallest of a[i], a[i+1], … a[n-1];

  swap a[i] with a[j];

  }

- **Example**:



Selection Sort

```
selectionSort( Comparable[ ] a ) {
   for ( int i = 0; i < a.length-1; i++ ) {
      int j = indexOfMin( i, a );
      swap elements at i and j
   }
}

indexOfMin(int start, Comparable[ ] a ) {
   Comparable min = a[start];
   int minIndex = start;
   for ( int i = start + 1; i < a.length; i++ )
      if ( a[i].compareTo( min ) < 0 ) {
         min = a[i];
         minIndex = i;
      }
   return minIndex;
}
```

indexOfMin: Returns the index of the smallest element of subarray a[start … length-1]

Using Selection Sort

- **Polymorphism**: Selection sort is polymorphic in the sense that it can be applied to any array whose elements implement the **Comparable** interface.
- **Example**:

```
Integer[ ] list1 = { new Integer( 8 ), new Integer( 6 ),
      /* blah, blah, blah… */   new Integer( 5 ) };
String[ ] list2 = { "Carol", "Bob", "Ted", "Alice", "Schultzie" };
selectionSort( list1 );
selectionSort( list2 );
```

Contents of list1:

3 5 6 8 11 15

Contents of list2:

Alice Bob Carol Schultzie Ted

Running time of Selection Sort

- **Efficiency**: How long does Selection Sort take to run?
- **What should we count**?
  - **Milliseconds of execution time**?
    - Depends on the speed of your particular **computer**
    - We prefer a **platform-independent measure**
  - **Statements of Java code that are executed**?
    - This depends on the **programmer**
    - We would like a quantity that is a function of the algorithm, not the specific way it was coded
  - **Number of times we call compareTo( )**?
    - This is an acceptable machine/programmer-independent statistic
    - This method is called every time through the innermost loop and **depends only on the algorithm**

Running time of Selection Sort

- **Running time depends on the contents of the array**:
  - **Length**: The principal determinant of running time is the number of elements, **n**, in the array
  - **Contents**: For some sorting algorithms, the running time may depend on the contents of the array. E.g., some algorithms run faster if the initial array is nearly sorted
  - **Worst-case running time**: Among all arrays of length **n**, consider the one that has the **highest** running time
  - **Average-case running time**: **Average** the running time over all arrays of length **n**. (This is very **messy**, so we won't do it.)
- **Worst-case running time**:
  - Let **T(n)** denote the time (measured as the number of calls to compareTo( )) required in the worst-case to sort an array of **n** items using Selection Sort

Running time of Selection Sort

- **How many times is compareTo( ) called**? Call this **T(n)**
  - Let **n** denote the length of array **n**
  - We go through the for-loop in selectionSort( ) exactly **n times**, for i = 0, 1, 2, …, n-1.
  - Each time we call indexOfMin(i, a), we call compareTo( ) to compare the min to each element of the **subarray a[i+1, …, n-1]**
  - In general, any subarray a[j, …, k] contains **k – j + 1 elements**
  - Thus, each call to indexOfMin(i, a) makes (n-1) - (i+1) + 1 = **n-i-1** calls to compareTo( )
  - To compute T(n), we simple add up **(n-i-1)**, for i = 0, 1, …, n-1

i=0	(n-0-1)	= n-1
i=1	(n-1-1)	= n-2
i=2	(n-2-1)	= n-3
…		
i=n-2	(n-(n-2)-1)	= 1
i=n-1	(n-(n-1)-1)	= 0

We rewrite the sum in increasing order

$$T(n) = 0 + 1 + \cdots + (n-3) + (n-2) + (n-1)$$
$$= \sum_{i=0}^{n-1} i$$

This summation notation is shorthand for the expression shown above.

Running time of Selection Sort

- **What is the value of this sum**?

  $T(n) = 0 + 1 + 2 + 3 + \ldots + (n-3) + (n-2) + (n-1)$

- **An old addition trick**: Group terms to get a common sum of values:

  $T(n) = 0 + 1 + 2 + 3 + \ldots + (n-3) + (n-2) + (n-1)$

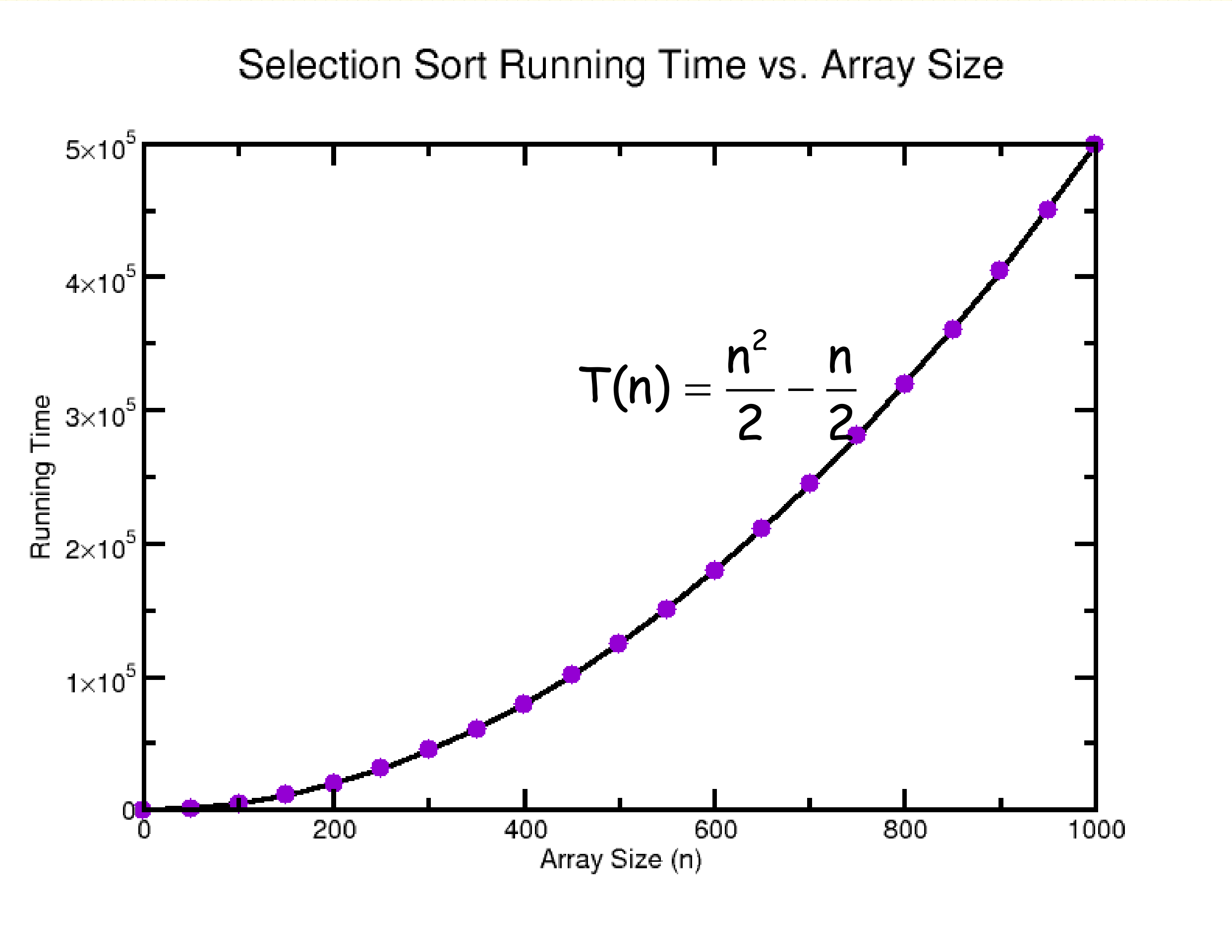
  Each of these pairs sums to n

  There are roughly **(n-1)/2** pairs, each of which sums to **n**. Thus, the total value is roughly n(n-1)/2. (In fact, this is exactly correct.)

- **Final running time**:

$$T(n) = \frac{n(n-1)}{2} = \frac{n^2 - n}{2} = \frac{n^2}{2} - \frac{n}{2}$$

Plot of Running Time vs. Array Size



Q: How efficient is Selection Sort?
A: It is pretty bad.

Big-"Oh" Notation (Revisited)

- Selection Sort's running time is given by the formula:

$$T(n) = \frac{n^2}{2} - \frac{n}{2}$$

- **Observation**: Efficiency is most critical for large **n**
  - As **n** becomes large, the quadratic $n^2/2$ term grows much more **rapidly** than the linear $n/2$ term
  - Small constant factors in running time like (1/2) depend on the **programmer's implementation**, and are best ignored
  - By ignoring the effects of
    - **lower order terms** (like n/2)
    - **constant factors** (like 1/2)

    we say that the running time grows **quadratically with n**, that is, "**on the order of n^2**", or succinctly **$O(n^2)$**

Big-"Oh" Notation

- "Big-Oh" Notation: a concise way of expressing the running time of an algorithm by ignoring less critical issues such as
  - **lower order (slower growing) terms** and
  - **constant multiplicative factors**
- Thus, the running time:

$$T(n) = \frac{n^2}{\underbrace{\boxed{2}}_{\text{Constant factor}}} - \overbrace{\boxed{\frac{n}{2}}}^{\text{Lower order term}}$$

- **$T(n) \rightarrow O(n^2)$**
- **Formal Mathematical Definition of "Big-Oh"**:
  - We will leave this for later courses (CMSC 250 and 351).