Problem 1. For this problem, assume that you have access to a procedure \texttt{Merge}(A, B, C, m, n), that merges two sorted arrays \(A[1..m]\) and \(B[1..n]\) into a single sorted array \(C[1..m+n]\). This procedure runs in \(m + n - 1\) comparisons. Assume we also have access to unlimited dynamic array allocation.

Suppose that you have a collection of \(n\), 1-dimensional arrays each of length \(n\), \(D[1..n][1..n]\), such that each array is sorted (i.e. \(D[i][j] \leq D[i][j+1]\)). We want to compute a single sorted array that contains all \(n^2\) elements.

The first strategy for accomplishing this is called \textit{cascading merging}. First merge \(D[1]\) with \(D[2]\) to form an array of length \(2n\), then merge this array with \(D[3]\) to form an array of length \(3n\), etc. The final merge is between an array of length \((n-1)n\) with \(D[n]\) to form the final sorted array of length \(n^2\).

The second strategy is called \textit{balanced merging}. Assume that \(n\) is a power of 2. First merge pairs \(D[1]\) and \(D[2]\) together, \(D[3]\) and \(D[4]\) together, and so on until merging \(D[n-1]\) and \(D[n]\) together. The result is \(n/2\) sorted arrays of size \(2n\) each. Next, repeat the balanced merging on these arrays, resulting in \(n/4\) arrays of size \(4n\) each. This is repeated until there is one array of size \(n^2\).

(a) What is the exact number of comparisons for cascading merging? Justify your answer. Give as simple a function \(g(n)\) as possible, such that the number of comparisons is \(\Theta(g(n))\).

(b) What is the exact number of comparisons for balanced merging? Justify your answer. Give as simple a function \(g(n)\) as possible, such that the number of comparisons is \(\Theta(g(n))\).

(c) Which algorithm is better, cascading merge or balanced merge (or is neither better)?

Problem 2. Assume merging two lists takes exactly \(\lceil m \log(4n/m) \rceil\) comparisons.

(a) What is the number of comparisons for cascading merging? Be as exact as reasonably possible. Justify your answer. Give as simple a function \(g(n)\) as possible, such that the number of comparisons is \(\Theta(g(n))\).

(b) What is the number of comparisons for balanced merging? Be as exact as reasonably possible. Justify your answer. Give as simple a function \(g(n)\) as possible, such that the number of comparisons is \(\Theta(g(n))\).