Problem 1. The Hamiltonian path problem (HP) is: Given an undirected graph $G = (V, E)$, does there exist a simple path that passes through every vertex of $G$? Show that the Hamiltonian path problem is in NP.

Problem 2. Assume that you have a polynomial time algorithm to determine if an undirected graph $G = (V, E)$ has a Hamiltonian path. Show that you can actually find a Hamiltonian path in polynomial time.

Problem 3. A Stable Set is a set of vertices in a graph that do not have any edges between them. Assume that the vertices of a graph have weights. The Weighted Stable Set Problem is to find a stable set of vertices whose sum of weights is as large as possible.

(a) Define a decision version of the weighted stable set problem.

(b) Show that the decision version of the weighted stable set problem is in NP.

(c) Show that if you could solve the optimization version in polynomial time that you could also solve the decision version in polynomial time.

(d) Challenge Problem. Show that if you could solve the decision version in polynomial time that you could also solve the optimization version in polynomial time. Note: There are two steps here. First find the value of maximum stable set, and then find the vertices themselves.