Asymptotic Notations.

\( \Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \} \).

\( O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0 \} \).

\( \Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0 \} \).

\( f(n) = o(g(n)) \text{ if } \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0. \)

\( f(n) = \omega(g(n)) \text{ if } \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty. \)

\( f(n) \sim g(n) \text{ if } f(n) = g(n) + o(g(n)). \)

Logarithms.

\[ \log_b a = \frac{\log_c a}{\log_c b} \quad a = b^{\log_b a} \quad \log_c(ab) = \log_c a + \log_c b \quad \log_b a^n = n \log_b a \]

\[ \log_b (1/a) = -\log_b a \quad \log_b a = \frac{1}{\log_b b} \quad a^{\log_b n} = n^{\log_b a} \]

Stirling’s Formula.

\[ n! \approx \left( \frac{n}{e} \right)^n \sqrt{2\pi n} \]

Recurrences “Master Theorem”:

\[ T(n) = \begin{cases} aT(n/b) + cn^d & n > 1 \\ f & n = 1 \end{cases} \]

implies

\[ T(n) = \begin{cases} (f + \frac{c}{ab^{-d-1}}) n^{\log_b a} - \frac{cn^d}{ab^{-d-1}} = \begin{cases} \Theta(n^{\log_b a}) & a > b^d \\ \Theta(n^d) & a < b^d \end{cases} \\ n^d(f + c \log_b n) = \Theta(n^d \log_b n) \end{cases} \]

Summing solutions: If

\[ T(n) = \begin{cases} aT(n/b) + \sum c_i n^{d_i} & n > 1 \\ f & n = 1 \end{cases} \]

then we can just sum the solutions of each recurrence:

\[ T_i(n) = \begin{cases} aT_i(n/b) + c_i n^{d_i} & n > 1 \\ 0 & n = 1 \end{cases} \]

and add in \( fn^{\log_b a} \) for the contribution from the leaves.
Summations.
Distribution law:
\[
\left( \sum_{i=1}^{m} a_i \right) \left( \sum_{j=1}^{n} b_j \right) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n} a_i b_j \right)
\]
Interchanging order of summation:
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij}
\]
Splitting range:
\[
\sum_{k=1}^{n} a_k = \sum_{k=1}^{r} a_k + \sum_{k=r+1}^{n} a_k
\]
Arithmetic series:
\[
\sum_{k=1}^{n} k = 1 + 2 + \ldots + n = \frac{n(n + 1)}{2}
\]
Geometric series:
\[
\sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{x^{n+1} - 1}{x - 1} \quad x \neq 1
\]
\[
\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x} \quad x < 1
\]
Harmonic series:
\[
H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k} = \ln n + O(1)
\]
Telescoping series:
\[
\sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0
\]
Products:
\[
\prod_{k=1}^{n} a_k = a_1 a_2 \cdots a_n \quad \log \prod_{k=1}^{n} a_k = \sum_{k=1}^{n} \log a_k
\]
Approximation by integrals:
\[
\int_{m-1}^{n} f(x)dx \leq \sum_{k=m}^{n} f(k) \leq \int_{m}^{n+1} f(x)dx \quad \text{for } f(x) \text{ monotonically increasing}
\]
\[
\int_{m}^{n+1} f(x)dx \leq \sum_{k=m}^{n} f(k) \leq \int_{m-1}^{n} f(x)dx \quad \text{for } f(x) \text{ monotonically decreasing}
\]
Quadratic Formula.
\[
a x^2 + b x + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
Probability.
\[
E[X] = \sum_{x} x \Pr\{X = x\} , \quad \text{Var}[X] = E[(X - E(X))^2] = E[X^2] - E^2[X] , \quad \sigma[X] = \sqrt{\text{Var}[X]} .
\]