Problem 1. Show that $4n^2 - 3n + 3 = \Omega(2n^2 + 5n - 5)$ using the definition of $\Omega$ given in class (do not use limits). Make your constants $c$ and $n_0$ reasonably small.

Problem 2.
(a) In what circumstances would you want to use a quadratic sorting algorithm rather than a $\Theta(n \log n)$ algorithm?
(b) Which quadratic sorting algorithm would use? Why?
(c) Give the code for your quadratic sorting algorithm.

Problem 3. Consider an array of size five with the numbers in the following order
40, 10, 20, 30, 50.
(a) Form the heap using the standard (Williams) algorithm. Show the heap as a tree. Show the heap as an array. Exactly how many comparisons did heap creation use?
(b) Start with the heap created in Part (a). Show the array after each element sifts down after heap creation. How many comparisons does each sift use? What is the total number of comparisons after heap creation?

Problem 4. Let $A[1, \ldots, n]$ be an array of $n$ numbers (some positive and some negative).
(a) Give an algorithm to find which two numbers have sum closest to zero. Make your algorithm as efficient as possible. Write it in pseudo-code.
(b) Analyze its running time.

Problem 5. Assume you are given a list of $n$ values, where you know that every value is within $k$ positions of its true sorted position. You can assume $k$ is “small”.
(a) Give a lower bound on the number of comparisons needed to sort the list as a function of $k$ and $n$.
(b) Give an efficient algorithm for sorting the list. Try to minimize the number of comparisons. Analyze how many comparisons your algorithm uses as a function of $k$ and $n$.
(c) Compare your upper and lower bounds.