Spanning Tree

- Set of edges connecting all nodes in graph
  - need $N-1$ edges for $N$ nodes
  - no cycles, can be thought of as a tree

- Can build tree during traversal

(a) Graph G
(b) Spanning tree T of graph G
Minimum Spanning Tree (MST)

Spanning tree with minimum total edge weight

(a) Graph G

(b) A spanning tree of cost $C = 43$

(c) A minimum spanning tree of cost $C = 28$
Minimum Spanning Tree (MST)

- Possible to have multiple MSTs
  - Different spanning trees with same weight

- Example applications
  - Minimize length of telephone lines for neighborhood
  - Minimize distance of airplane routes serving cities
Three well known algorithms

1. **Borůvka’s algorithm** [1926]
   - For constructing efficient electricity network
   - Rediscovered by Sollin in 1960s

2. **Prim’s algorithm** [1957]
   - First discovered by Vojtěch Jarník in 1930
   - Similar to Djikstra’s algorithm

3. **Kruskal’s algorithm** [1956]
   - By Prof. Clyde Kruskal’s uncle
MST – Kruskal’s Algorithm

sort edges by weight (from least to most)

\[ \text{tree} = \emptyset \]

for each edge \((X,Y)\) in order

\[
\text{if it does not create a cycle}\\
\text{add} (X,Y) \text{ to tree}\\
\text{stop when tree has } N-1 \text{ edges}
\]

Keeps track of

- lightest edge remaining
- whether adding edge to MST creates cycle

Optimal solution computed with greedy algorithm
MST – Kruskal’s Algorithm Example