CMSC 132: Object-Oriented Programming II

Sorting

Department of Computer Science
University of Maryland, College Park
Overview

- **Comparison sort**
  - Bubble sort
  - Selection sort
  - Tree sort
  - Heap sort
  - Quick sort
  - Merge sort
  \[O(n^2)\]

- **Linear sort**
  - Counting sort
  - Bucket (bin) sort
  - Radix sort
  \[O(n)\]
Sorting

Goal

- Arrange elements in predetermined order
  - Based on key for each element
- Derived from ability to compare two keys by size

Properties

- Stable $\Rightarrow$ relative order of equal keys unchanged
  - Stable: $3, 1, 4, 3, 3, 2 \rightarrow 1, 2, 3, 3, 3, 4$
  - Unstable: $3, 1, 4, 3, 3, 2 \rightarrow 1, 2, 3, 3, 3, 4$
- In-place $\Rightarrow$ uses only constant additional space
- External $\Rightarrow$ can efficiently sort large # of keys
Sorting

Comparison sort
- Only uses pairwise key comparisons
- Proven lower bound of $O(n \log(n))$

Linear sort
- Uses additional properties of keys
**Bubble Sort**

**Approach**
1. Iteratively sweep through shrinking portions of list
2. Swap element $x$ with its right neighbor if $x$ is larger

**Performance**
- $O(n^2)$ average / worst case
Bubble Sort Example

Sweep 1

7 2 8 5 4
2 7 8 5 4
2 7 8 5 4
2 7 8 5 4
2 7 5 8 4
2 7 5 4 8

Sweep 2

2 7 5 4 8
2 7 5 4 8
2 7 5 4 8
2 5 7 4 8
2 5 4 7 8
2 5 4 7 8
2 4 5 7 8

Sweep 3

2 5 4 7 8
2 5 4 7 8
2 5 4 7 8
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8

Sweep 4

2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
void bubbleSort(int[] a) {
    int outer, inner;
    for (outer = a.length - 1; outer > 0; outer--)
        for (inner = 0; inner < outer; inner++)
            if (a[inner] > a[inner + 1])
                swap(a, inner, inner+1);
}

void swap(int a[], int x, int y) {
    int temp = a[x];
    a[x] = a[y];
    a[y] = temp;
}
Selection Sort

Approach
1. Iteratively sweep through shrinking portions of list
2. Select smallest element found in each sweep
3. Swap smallest element with front of current list

Performance
- $O(n^2)$ average / worst case

Example
void selectionSort(int[] a) {
    int outer, inner, min;
    for (outer = 0; outer < a.length - 1; outer++) {
        min = outer;
        for (inner = outer + 1; inner < a.length; inner++) {
            if (a[inner] < a[min]) {
                min = inner;
            }
        }
        swap(a, outer, min);
    }
}
**Tree Sort**

**Approach**
1. Insert elements in binary search tree
2. List elements using *inorder* traversal

**Performance**
- Binary search tree
  - $O(n \log(n))$ average case
  - $O(n^2)$ worst case
- Balanced binary search tree
  - $O(n \log(n))$ average / worst case

**Example**

```
{ 7, 2, 8, 5, 4 }
```

Binary search tree
Heap Sort

**Approach**
1. Insert elements in heap
2. Remove smallest element in heap, repeat
3. List elements in order of removal from heap

**Performance**
- $O(n \log(n))$ average / worst case

**Example**

Heap

{ 7, 2, 8, 5, 4 }
Quick Sort

Approach
1. Select pivot value (near median of list)
2. Partition elements (into 2 lists) using pivot value
3. Recursively sort both resulting lists
4. Concatenate resulting lists

For efficiency pivot needs to partition list evenly

Performance
- $O(n \log(n))$ average case
- $O(n^2)$ worst case
Quick Sort Algorithm

1. If list below size K
   - Sort w/ other algorithm

2. Else pick pivot $x$ and partition $S$ into
   - L elements $< x$
   - E elements $= x$
   - G elements $> x$

3. Quicksort L & G

4. Concatenate L, E & G
   - If not sorting in place
void quickSort(int[] a, int x, int y) {
    int pivotIndex;
    if ((y - x) > 0) {
        pivotIndex = partionList(a, x, y);
        quickSort(a, x, pivotIndex - 1);
        quickSort(a, pivotIndex+1, y);
    }
}

int partionList(int[] a, int x, int y) {
    ... // partitions list and returns index of pivot
}
Quick Sort Example

Partition & Sort

Result
```c
int partitionList(int[] a, int x, int y) {
    int pivot = a[x];
    int left = x;
    int right = y;
    while (left < right) {
        while ((a[left] < pivot) && (left < right))
            left++;
        while (a[right] > pivot)
            right--;
        if (left < right)
            swap(a, left, right);
    }
    swap(a, x, right);
    return right;
}
```
Merge Sort

Approach

1. Partition list of elements into 2 lists
2. Recursively sort both lists
3. Given 2 sorted lists, merge into 1 sorted list
   a) Examine head of both lists
   b) Move smaller to end of new list

Performance

- $O(n \log(n))$ average / worst case
Merge Example
Merge Sort Example
void mergeSort(int[] a, int x, int y) {
    int mid = (x + y) / 2;
    if (y == x) return;
    mergeSort(a, x, mid);
    mergeSort(a, mid+1, y);
    merge(a, x, y, mid);
}

void merge(int[] a, int x, int y, int mid) {
    ... // merges 2 adjacent sorted lists in array
}
void merge (int[] a, int x, int y, int mid) {
    int size = y - x;
    int left = x;
    int right = mid + 1;
    int[] tmp; int j;
    for (j = 0; j < size; j++) {
        if (left > mid) tmp[j] = a[right++];
        else if (right > y) || (a[left] < a[right])
            tmp[j] = a[left++];
        else tmp[j] = a[right++];
    }
    for (j = 0; j < size; j++)
        a[x+j] = tmp[j];
}
Counting Sort

Approach

1. Sorts keys with values over range 0..\( k \)
2. Count number of occurrences of each key
3. Calculate \# of keys \( \leq \) each key
4. Place keys in sorted location using \# keys counted
   - If there are \( x \) keys \( \leq \) key \( y \)
   - Put \( y \) in \( x^{th} \) position
   - Decrement \( x \) in case more instances of key \( y \)

Properties

- \( O( n + k ) \) average / worst case
Counting Sort Example

- Original list
  \[
  \begin{array}{cccccc}
  7 & 2 & 8 & 5 & 4 \\
  \end{array}
  \]

- Count
  \[
  \begin{array}{cccccccc}
  0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
  \end{array}
  \]

- Calculate # keys \leq value
  \[
  \begin{array}{cccccccc}
  0 & 0 & 1 & 1 & 2 & 3 & 3 & 4 & 5 \\
  \end{array}
  \]
Counting Sort Example

Assign locations

7 2 8 5 4

4-1 = 3

2 7

0 1 2 3 4

7 2 8 5 4

5-1 = 4

7 2 8 5 4

5-1 = 4

7 2 8 5 4

2-1 = 1

2 4 5 7 8

0 1 2 3 4

1-1 = 0

2 7

0 1 2 3 4

7 2 8 5 4

3-1 = 2

2 5 7 8

0 1 2 3 4
void countSort(int[] a, int k) {
  int[] b; int[] c; int i;
  for (i = 0; i ≤ k; i++) { // initialize counts
    c[i] = 0;
  }
  for (i = 0; i < a.size(); i++) { // count # keys
    c[a[i]]++;
  }
  for (i = 1; i ≤ k; i++) { // calculate # keys ≤ value i
    c[i] = c[i] + c[i - 1];
  }
  for (i = a.size() - 1; i > 0; i--) {
    b[c[a[i]] - 1] = a[i]; // move key to location
    c[a[i]]--;
  }
  for (i = 0; i < a.size(); i++) { // copy sorted list back to a
    a[i] = b[i];
  }
}
Bucket (Bin) Sort

Approach
1. Divide key interval into $k$ equal-sized subintervals
2. Place elements from each subinterval into bucket
3. Sort buckets (using other sorting algorithm)
4. Concatenate buckets in order

Properties
- Pick large $k$ so can sort $n / k$ elements in $O(1)$ time
- $O(n)$ average case
- $O(n^2)$ worst case
- If most elements placed in same bucket and sorting buckets with $O(n^2)$ algorithm
Bucket Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231

2. Bucket based on 1st digit, then sort bucket
   - 192, 144, 152  \rightarrow 144, 152, 192
   - 253, 231  \rightarrow 231, 253
   - 552  \rightarrow 552
   - 623  \rightarrow 623
   - 752  \rightarrow 752

3. Concatenate buckets
   - 144, 152, 192 231, 253 552 623 752
Radix Sort

Approach

1. Decompose key $C$ into components $C_1, C_2, \ldots, C_d$
   - Component $d$ is least significant
   - Each component has values over range $0..k$
2. For each key component $i = d$ down to $1$
   - Apply linear sort based on component $C_i$
     (sort must be stable)

Example key components
- Letters (string), digits (number)

Properties
- $O(d \times (n+k)) \approx O(n)$ average / worst case
Radix Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231

2. Sort on 3\textsuperscript{rd} digit (counting sort from 0-9)
   - 231, 192, 152, 752, 552, 623, 253, 144

3. Sort on 2\textsuperscript{nd} digit (counting sort from 0-9)
   - 623, 231, 144, 152, 752, 552, 253, 192

4. Sort on 1\textsuperscript{st} digit (counting sort from 0-9)
   - 144, 152, 192, 231, 253, 552, 623, 752

Compare with: counting sort from 144-752
## Sorting Properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Comparison Sort</th>
<th>Avg Case Complexity</th>
<th>Worst Case Complexity</th>
<th>In Place</th>
<th>Can be Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble</td>
<td>✓</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Selection</td>
<td>✓</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Tree</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n^2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heap</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Merge</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Counting</td>
<td></td>
<td>$O(n)$</td>
<td>$O(n)$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Bucket</td>
<td></td>
<td>$O(n)$</td>
<td>$O(n^2)$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Radix</td>
<td></td>
<td>$O(n)$</td>
<td>$O(n)$</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
Sorting Summary

- Many different sorting algorithms
- Complexity and behavior varies
- Size and characteristics of data affect algorithm