Trees & Binary Search Trees

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Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements
- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes
Trees

• Terminology
  • Root ⇒ node with no parent
  • Leaf ⇒ all nodes with no children
  • Interior ⇒ all nodes with children
Trees

• Terminology
  • Sibling ⇒ node with same parent
  • Descendent ⇒ children nodes & their descendents
  • Subtree ⇒ portion of tree that is a tree by itself
    ⇒ a node and its descendents
Trees

- Terminology
  - Level ⇒ is a measure of a node’s distance from root
  - Definition of level
    - If node is the root of the tree, its level is 1
    - Else, the node’s level is 1 + its parent’s level
  - Height (depth) ⇒ max level of any node in tree

\[ \text{Height} = 3 \]
Binary Trees

- Binary tree
  - Tree with 0–2 children per node
    - Left & right child / subtree
Tree Traversal

• Often we want to
  • Find all nodes in tree
  • Determine their relationship

• Can do this by
  • Walking through the tree in a prescribed order
  • Visiting the nodes as they are encountered

• Process is called tree traversal
Tree Traversal

- **Goal**
  - Visit every node in binary tree

- **Approaches**
  - **Depth first**
    - Preorder => parent before children
    - Inorder => left child, parent, right child
    - Postorder => children before parent
  - **Breadth first** => closer nodes first
Tree Traversal Methods

- **Pre-order**
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree

- **In-order**
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively right subtree

- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last
Tree Traversal Methods

• Breadth-first

BFS(Node n) {
    Queue Q = new Queue()
    Q.enqueue(n); // insert node into Q
    while ( !Q.empty() ) {
        n = Q.dequeue(); // remove next node
        if ( !n.isEmpty() ) {
            visit(n); // visit node
            Q.enqueue(n.Left()); // insert left subtree in Q
            Q.enqueue(n.Right()); // insert right subtree in Q
        }
    }
}
Tree Traversal Examples

• Pre-order (prefix)
  • $+ \times 2 \ 3 \ / \ 8 \ 4$

• In-order (infix)
  • $2 \times 3 + 8 / 4$

• Post-order (postfix)
  • $2 \ 3 \times 8 \ 4 \ / \ +$

• Breadth-first
  • $+ \times / \ 2 \ 3 \ 8 \ 4$

Expression tree
Binary Tree Implementation

• Using a class to represent a Node
  Class Node {
    KeyType key;
    Node left, right; // null if empty
  }

  Node root = null; // Empty Tree

• Using a Polymorphic Binary Tree
  • We will talk about this implementation later on
Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = $O(n)$
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = $O(\log(n))$
  - $2^{\text{Height}} - 1 = n$ (# of nodes)
  - Useful for searches

Degenerate binary tree

Balanced binary tree
Binary Search Trees

- Key property
  - Value at node
    - Smaller values in left subtree
    - Larger values in right subtree
- Example
  - $Y > X$
  - $Y < Z$
Binary Search Trees

- Examples

Binary search trees

Non-binary search tree
Tree Traversal Examples

• Pre-order
  • 44, 17, 32, 78, 50, 48, 62, 88

• In-order
  • 17, 32, 44, 48, 50, 62, 78, 88

Post-order
  32, 17, 48, 62, 50, 88, 78, 44

Breadth-first
  44, 17, 78, 32, 50, 88, 48, 62

Sorted order!

Binary search tree
Example Binary Searches

• Find (2)

```
2 < 10, left
2 < 5, left
2 = 2, found
2 < 5, left
2 = 2, found
```
Example Binary Searches

• Find (25)

25 > 10, right
25 < 30, left
25 = 25, found

25 > 5, right
25 < 45, left
25 < 30, left
25 > 10, right
25 = 25, found
Binary Search Properties

- Time of search
  - Proportional to height of tree
  - Balanced binary tree
    - \( O(\log(n)) \) time
  - Degenerate tree
    - \( O(n) \) time
    - Like searching linked list / unsorted array
- Requires
  - Ability to compare key values
Binary Search Tree Construction

• How to build & maintain binary trees?
  • Insertion
  • Deletion
• Maintain key property (invariant)
  • Smaller values in left subtree
  • Larger values in right subtree
Binary Search Tree – Insertion

• Algorithm
  1. Perform search for value X
  2. Search will end at node Y (if X not in tree)
  3. If X < Y, insert new leaf X as new left subtree for Y
  4. If X > Y, insert new leaf X as new right subtree for Y

• Observations
  • O( log(n) ) operation for balanced tree
  • Insertions may unbalance tree
Example Insertion

- Insert (20)

20 > 10, right
20 < 30, left
20 < 25, left
Insert 20 on left
Binary Search Tree – Deletion

• Algorithm
  1. Perform search for value X
  2. If X is a leaf, delete X
  3. Else  // must delete internal node
    a) Replace with largest value Y on left subtree
       OR smallest value Z on right subtree
    b) Delete replacement value (Y or Z) from subtree

• Observation
  • O( log(n) ) operation for balanced tree
  • Deletions may unbalance tree
Example Deletion (Leaf)

- Delete (25)

25 > 10, right
25 < 30, left
25 = 25, delete
Example Deletion (Internal Node)

- Delete (10)

Replacing 10 with largest value in left subtree

Replacing 5 with largest value in left subtree

Deleting leaf
Example Deletion (Internal Node)

- Delete (10)

Replacing 10 with smallest value in right subtree

Deleting leaf

Resulting tree
Building Maps w/ Search Trees

- Binary Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type <K extends Comparable<K>>
    - Denotes any type K that can be compared to K’s
BST (Binary Search Tree) Implementation

- Implementing Tree using traditional approach
- Based on the BST definition below let’s see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)

```java
public class BinarySearchTree <K extends Comparable<K>, V> {
    private class Node {
        private K key;
        private V data;
        private Node left, right;
        public Node(K key, V data) {
            this.key = key;
            this.data = data;
        }
    }
    private Node root;
}
```

- See code distribution BinaryTreeCode.zip
BST Testing

• How can we test the correctness of BST Methods?
• What is the best approach?