CMSC 351: Practice Questions for Final Exam

These are practice problems for the upcoming final exam, which we will go over in class on Monday. You will be given a sheet of notes for the exam. Also, go over your homework assignments. **Warning:** This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

**Problem 1.** Assume you are given a list of \( n \) values, where you know that every value is within \( k \) positions of its true sorted position. You can assume \( k \) is “small”.

(a) Give a lower bound on the number of comparisons needed to sort the list as a function of \( k \) and \( n \).

(b) Give an efficient algorithm for sorting the list. Try to minimize the number of comparisons. Analyze how many comparisons your algorithm uses as a function of \( k \) and \( n \).

(c) Compare your upper and lower bounds.

**Problem 2.** Assume that you developed an algorithm to find the (index of the) \( n/3 \) smallest element of a list of \( n \) elements in \( 2n \) comparisons.

(a) Using the algorithm (as a black box), give an algorithm, efficient in the worst case, to find the \( k \)th smallest element of a list.

(b) Write down a recurrence for (a bound on) the number of comparisons it executes in the worst case.

(c) Solve the recurrence (using constructive induction). Find the high order term exactly (but you do not need any low order terms).

(d) Using the (black box) algorithm for finding the \( n/3 \) smallest element and using the ideas and results of Parts (a), (b), and (c), give an efficient algorithm to find (the index of) two elements, the \( k_1 \)th smallest and the \( k_2 \) smallest (for inputs \( k_1 \) and \( k_2 \)). The algorithm description can be very high level and brief.

(e) How many comparisons does it use? Find the high order term exactly (but you do not need any low order terms). Give a brief justification.
Problem 3. A graph is tripartite if the vertices can be partitioned into three sets so that there are no edges internal to any set. The complete tripartite graph, $K(a,b,c)$, has three sets of vertices with sizes $a$, $b$, and $c$ and all possible edges between each pair of sets of vertices. $K(3,2,3)$ is pictured below. A Hamiltonian cycle in a graph is a cycle that traverses every vertex exactly once.

(a) For which values of $n$ does $K(1,1,n)$ have a Hamiltonian cycle. Justify your answer.
(b) For which values of $n$ does $K(1,n,n)$ have a Hamiltonian cycle. Justify your answer.
(c) For which values of $n$ does $K(n,n,n)$ have a Hamiltonian cycle. Justify your answer.

Problem 4. Let $G = (V,E)$ be an undirected graph. A triangle is a set of three vertices such that each pair has an edge.

(a) Give an efficient algorithm to find all of the triangles in a graph.
(b) How fast is your algorithm?

Problem 5. This problem is more open-ended than you would see on an exam: If you do not know how to play Sudoku, look it up. Normally, Sudoku is played on a $9 \times 9$ grid.

(a) Generalize Sudoku to larger grids.
(b) State the (generalized) Sudoku puzzle as a decision problem.
(c) Show that the (generalized) Sudoku decision problem is in $\text{NP}$. 
(d) Show that if you can solve the (generalized) decision problem in polynomial time that you can solve a (generalized) Sudoku puzzle in polynomial time.

Problem 6. Construct the string-matching automaton for the pattern $P = aabab$ and illustrate its operation on the text string $T = aaababaababaababab$. 

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