CMSC 423 Homework #1:
Due: Sep. 22 at the start of class

You may discuss these problems with your classmates, but you must write up your solutions independently, without using common notes or worksheets. You must indicate at the top of your homework who you worked with. Your write up should be clear, concise, and neat. You are trying to convince a skeptical reader that your answers are correct. Messy or hard-to-read homeworks will not be graded.

1. Consider the strings $a = \text{"nonsense"}$ and $b = \text{"oneness"}$. Suppose that a gap costs 5, a match cost $-1$, and the cost of aligning two mismatching characters $x$ and $y$ is given by the function $\text{cost}$:

   \[
   \begin{align*}
   \text{cost}(n,o) &= \text{cost}(n,s) = \text{cost}(n,e) = 2 \\
   \text{cost}(o,e) &= \text{cost}(o,s) = \text{cost}(s,e) = 1
   \end{align*}
   \]

   (a) Provide a well-labeled dynamic programming table for the global alignment between $a$ and $b$. Show backtracking arrows. Find the optimal, lowest-cost alignment and its cost.

   (b) Provide a well-labeled dynamic programming table for the local alignment between $a$ and $b$. Show backtracking arrows. Find the optimal alignment and its cost.

2. The Shortest Common Supersequence between strings $x$ and $y$ is the shortest string $z$ such that both $x$ and $y$ are subsequences of $z$. ($x$ is a “subsequence” of $z$ if the letters of $x$ appear in order in $z$, not necessarily consecutively: at is a subsequence of act).

   By modifying the basic sequence alignment algorithm, give an efficient algorithm to find the shortest common supersequence for two sequences $x$ and $y$.

3. You run an ice cream business, and you want to place some advertisements in your local newspaper. There are two kinds of ads you can run, and you’ve noticed that Type-C works best on cold days (by promoting the good taste of your ice cream) and Type-W works best on warm days (by mentioning how cold and refreshing your ice cream is). Depending on the weather and which ad you run, you see a certain amount of increased profit that day:

   \[
   \begin{array}{c|cc}
   & \text{Cold} & \text{Warm} \\
   \hline
   \text{Type-C ad} & +$75 & +$50 \\
   \text{Type-W ad} & +$50 & +$100 \\
   \end{array}
   \]

   You have committed to running an ad every day. The cost of placing either a Type-C or Type-W ad is $10 per day. But the newspaper charges you a fee of $25 every time you change which ad you are running.

   You are given a (perfectly correct) weather prediction for the next $n$ days. Design a dynamic programming algorithm to select which ad to run on each of the next $n$ days to maximize your profit.

   Examples:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>WWWCCWCCWCCW</td>
<td>WWWCCWCCWCCW</td>
<td>$895</td>
</tr>
<tr>
<td>WWWCWCCWCCW</td>
<td>WWWCWCCWCCW</td>
<td>$220</td>
</tr>
<tr>
<td>WWWWWW</td>
<td>WWWWWW</td>
<td>$530</td>
</tr>
</tbody>
</table>