Problem 1. Use mathematical induction to show that

(a) \[ \sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3} \]

(b) \[ \sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \]

Problem 2. See bottom of this sheet.

(a) Assume \( b^x = a \). What is \( x \) (in terms of \( a \) and \( b \))?

(b) Using only part (a), show that \( \log_c(ab) = \log_c a + \log_c b \).

(c) Show that \( a^{\log_b n} = n^{\log_b a} \)

Problem 3. Differentiate the following functions:

(a) \( \ln(x^2 + 5) \)

(b) \( \lg(x^2 + 5) \)

(c) \( \frac{1}{\ln(x^2 + 5)} \)

Problem 4. Integrate the following functions:

(a) \( \frac{1}{x} \)

(b) \( \frac{1}{\ln(x^2 + 7)} \)

(c) \( \ln x \) [HINT: Use integration by parts.]

(d) \( x \ln x \) [HINT: Use integration by parts.]

(e) \( x \lg x \)

\[
\begin{align*}
\lg n & = \log_2 n \\
\ln n & = \log_e n \\
\lg^k n & = (\lg n)^k \\
\lg \lg n & = \lg(\lg n)
\end{align*}
\]

For all real \( a > 0, b > 0, c > 0 \), and \( n \),

\[
\begin{align*}
a & = b^{\log_b a} \\
\log_c (ab) & = \log_c a + \log_c b \\
\log_b a^n & = n \log_b a \\
\log_b a & = \frac{\log_c a}{\log_c b} \\
\log_b (1/a) & = -\log_b a \\
\log_b a & = \frac{1}{\log_a b} \\
a^{\log_b n} & = n^{\log_b a}
\end{align*}
\]