Lecture 18
Parallelizing Algorithms
Recall

• Concurrency: multiple flows of control
• Parallelism: simultaneous flows of control
• Reasons for concurrency
  – Improvements in throughput, responsiveness
  – Natural fit to application domain
• Reasons for parallelism
  – Performance improvements
Parallelizing Algorithms

• One important topic in parallelism: making existing sequential algorithms run in parallel
  – Existing algorithms often perform common, important tasks (e.g. sorting, searching, depth-first search)
  – Making them more efficient improves application / system performance
• Tasks offer a framework for studying parallelization
  – Idea: identify computations within algorithms that can be thought of as tasks (i.e. can be performed independently)
  – Execution these tasks concurrently, BUT ...
  – ... Tune concurrent execution to make best use of computational resources
Loop Parallelization

- Many sequential algorithms are *iterative* (i.e. use loops)
- Loop parallelization: perform (groups of) iterations in parallel
  - Sequential
    ```java
    for (Element e : collection)
    process(e);
    ```
  - Parallel
    ```java
    for (Element e : collection)
    exec.execute (new Runnable() {
        public void() run {
            process(e);
        }
    });
    ```
- When does this work?
  - Iterations must be independent (i.e. result of one iteration does not depend on the other)
  - Example: adding 1 to each element in an array
    - The result of processing each element is independent of the others
    - They can be made into tasks!
Loop Parallelization (2)

- **Variation:** grouping several iterations together into tasks
  - Consider summing elements in an array
    
    ```java
    sum = 0;
    for (int i=0; i < a.length; i++)
        sum += a[i];
    ```
  - Iterations are not independent and cannot be made into tasks “as is”
  - However, several tasks can be created!
    ```java
    int sum[] = new int[NUMTASKS]; // Ensure initialization to 0
    for (int i=0; i < NUMTASKS; i++) {
        exec.execute (new Runnable() {
            public void run () {
                for (j = i*NUMTASKS; j < (i+1)*NUMTASKS; j++)
                    sum[i] += a[j];
            }
        });
    }
    ```
    // After termination, sum up sum[i] values
  - **In this case, independent tasks created**
    - However, final result depends on collecting results of tasks
    - This requires determining when tasks have terminated!
    - Several ways to do this: termination of executor, barriers, latches, etc.
  - **You still have to worry about thread-safety, visibility!!**
Parallelizing Recursion

• Sometimes algorithms are recursive
  Example: depth-first search of a tree
    • Process node
    • Search each subtree
    • If there are no subtrees, return

• Similar ideas to loop parallelization can be used
  – Turn recursive calls into tasks!
  – Execute tasks concurrently
  – Considerations
    • Tasks should be independent
    • Works best if algorithms are **tail-recursive**: recursive calls issued at end
Example (JCIP pp. 182): Depth-First Search

- **Tree**: object in `List<Node<T>>`
  - List has only one node (the root)
  - Node methods
    - `getChildren()`: return list of subtrees (list of nodes)
    - `compute()`: perform computation on node

- **Sequential tail-recursive version**
  ```java
  public<T> void sequentialRecursive(List<Node<T>> nodes, Collection<T> results) {
    for (Node<T> n : nodes) {
      results.add(n.compute());
      sequentialRecursive(n.getChildren(), results);
    }
  }
  ```
  - Final operation of any call to `sequentialRecursive()` is the recursive call
  - This operation is therefore tail-recursive
Parallelizing Depth-First Search

- Task launching

  ```java
  public <T> void parallelRecursive(final Executor exec,
                                    List<Node<T>> nodes,
                                    final Collection<T> results) {
      for (final Node<T> n : nodes) {
        exec.execute(new Runnable() {
          public void run() { results.add(n.compute()); }
        });
        parallelRecursive(exec, n.getChildren(), results);
      }
  }
  ```

- Result collection

  ```java
  public <T> Collection<T> getParallelResults(List<Node<T>> nodes)
  throws InterruptedException {
    ExecutorService exec = Executors.newCachedThreadPool();
    Queue<T> resultQueue = new ConcurrentLinkedQueue<T>();
    parallelRecursive(exec, nodes, resultQueue);
    exec.shutdown();
    exec.awaitTermination();
    return resultQueue;
  }
  ```
Performance Tuning

• The previous examples showed how task boundaries can be defined for parallelization
• However: should every task be run concurrently?
  – There is overhead in task launching
    • Insertion into work queue
    • Retrieval from work queue by worker thread
  – There is only run-time benefit if the final run-time decreases!
• We will study this issue in the context of parallel sorting
Recall Quicksort

• A fast sequential sorting algorithm invented by Tony Hoare (Turing Award winner) based on
  – Partitioning
  – Recursion

• quickSortSegment (elts, i, j) sorts elements in segment of array elts starting at i and extending j elements to the right
  – First, partition segment into two subsegments: those less than elts[i] and those greater than elts[i]
    • elt[i] is called the pivot
    • Partitioning involves scanning through segment and potentially swapping pivot with other elements
  – Then, recursively sort each of the subsegments
Sequential Quicksort code from IntArraySortUtils.java

```java
public static void quickSortSegment (int[] elts, int first, int size) {
    if (size == 2) {
        if (elts[first] > elts[first+1])
            swap (elts, first, first+1);
    } else if (size > 2) {
        int pivotPosition = partitionSegment(elts, first, size);
        quickSortSegment (elts, first, pivotPosition-first);
        quickSortSegment (elts, pivotPosition+1, first+size-1-pivotPosition);
    }
}
```

- (Almost) tail-recursive!
- Since recursive calls work on disjoint parts of the array, these can be made parallel
- To parallelize, can try turning each recursive call into a task
Parallelized Quicksort Code from ParallelQuickSort.java (1)

- Task definition

```java
private class PQSTask implements Runnable {

    private final int[] elts;
    private final int first;
    private final int size;
    private final CountDownLatch numUnsorted;

    PQSTask (int[] elts, int first, int size, CountDownLatch numUnsorted) { ... }

    public void run () {
        if (size == 1) numUnsorted.countDown();
        else if (size == 2) {
            if (elts[first] > elts[first+1])
                IntArraySortUtils.swap (elts, first, first+1);
            reduceUnsorted(size); // Used for termination determination
        } else if (size > 2) {
            final int pivotPosition = IntArraySortUtils.partitionSegment(elts, first, size);
            numUnsorted.countDown(); // pivot in correct sorted position
            ThreadPool.execute(new PQSTask (elts, first, pivotPosition-first, numUnsorted));
            ThreadPool.execute(new PQSTask (elts, pivotPosition+1, first+size-1-pivotPosition, numUnsorted));
        }
    }
}
```
Parallelized Quicksort Code from ParallelQuickSort.java (2)

- **Sort driver**

  ```java
  public void sort (int[] elts) {
      int length = elts.length;
      CountDownLatch numUnsorted = new CountDownLatch(length);
      threadPool.execute(new PQSTask (elts, 0, length, numUnsorted));
      try {
          numUnsorted.await();
      } catch (InterruptedException e) {} 
  }
  ```
Performance

- Parallelized Quicksort is slower (on my two-core machine) than sequential Quicksort!
  - When sorting $k$ elements, on average $k/2$ tasks will be created
    - $k = 10$: 5 tasks
    - $k = 1,000,000$: 500,000 tasks!
  - The overhead of task management overwhelms the gains from parallelism

- Can solve this by coarsening task boundaries (fewer, bigger tasks)
Tuning Parallel Quicksort

• Several different ways to approach this
  – Key point: want to limit number of tasks based on number of CPUs
  – One idea:
    • Determine number of threads to be used
    • Determine size of sorting problem that should be handled sequentially, based on number of threads
    • Only create new tasks when the sorting problem they are handed is bigger than this limit

• How to determine number of threads?
  – Recall formula: \( N_{\text{threads}} = N_{\text{CPU}} \times U_{\text{CPU}} \times (1 + W/C) \)
  – For sorting, \( W/C \) is (very) low
  – In this case, \( N_{\text{threads}} = N_{\text{CPU}} + 1 \) (or 2) is a good idea

• How to determine sequential task limit?
  – If sorting \( k \) elements ...
  – ... set size limit to \( k / N_{\text{threads}} \)
  – E.g.: if \( k \) is 1,000, \( N_{\text{threads}} = 3 \), then sequential task limit is 333
    • If sorting \( \leq 333 \) elements, do so sequentially
    • Otherwise, do so in parallel
Tuned Parallelized Quicksort Code: ParallelQuickSortTunable.java

- Task is redefined:
  ```java
  public void run () {
      int sequentializeTarget = elts.length / (numCPUs+2);
      if (size <= sequentializeTarget) {
          sequentialSort.sortSegment (elts, first, size);
          reduceUnsorted (size);
      } else {
          final int pivotPosition = IntArraySortUtils.partitionSegment(...);
          final int size1 = pivotPosition-first;
          final int size2 = first+size-1-pivotPosition;
          numUnsorted.countDown(); // pivot in correct sorted position
          threadPool.execute(new PQSTask (elts, first, size1, numUnsorted));
          threadPool.execute(new PQSTask (elts, pivotPosition+1, size2, numUnsorted));
      }
  }
  ```
- Result: parallelized implementation is faster than sequential, in general