Instructions: Submit one writeup per group; please discuss within your group – other resources including the Web are not allowed for consultation. Write your solutions neatly; write only your group ID, not your names.

1. Suppose $P$ is a polytope in $n$ dimensions, and that $x$ is a point in $P$. Suppose the linear span of the set of tight constraints at $x$ has dimension smaller than $n$. Prove that $x$ is not a vertex. (Hint: You need to find a vector $r \neq 0$ and a scalar $\epsilon > 0$ such that $x + \epsilon r$, $x - \epsilon r \in P$. How will you find the requisite $r$?)


3. Given an undirected graph $G$ and some integer $t$, suppose we wish to find a smallest set of vertices $S$ such that at least $t$ edges in $G$ have the property that at least one of their end-points is in $S$. Let $\|v\|$ denote the usual 2-norm $\sqrt{\sum_{i=1}^{n} v_i^2}$ of a vector $v = (v_1, v_2, \ldots, v_n)$. Prove that the following semidefinite program is a valid relaxation for this problem (i.e., for every feasible $S$, there is a solution to this SDP):

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i=1}^{n} \frac{1 + v_0 \cdot v_i}{2} \\
& \quad \sum_{(i,j) \in E} \frac{3 + v_0 \cdot v_j + v_0 \cdot v_i - v_i \cdot v_j}{4} \geq t \\
& \quad \frac{3 + v_0 \cdot v_j + v_0 \cdot v_i - v_i \cdot v_j}{4} \leq 1, \quad (i, j) \in E \\
& \quad v_0 \cdot v_j + v_0 \cdot v_i + v_i \cdot v_j \geq -1, \quad (i, j) \in E \\
& \quad v_i \in \mathbb{R}^n, \|v_i\| = 1, \quad i \in (\{0\} \cup V)
\end{align*}
\]