

CMSC 330: Organization of Programming Languages

Examples of REs & Finite Automata

Describing Regular Expressions

- a) $0(0|1)^*0$
 - All strings beginning and ending in 0
- b) $((\epsilon | 0)1^*)^*$
 - All strings
- c) $(0|1)^*0(0|1)(0|1)$
 - All strings with 0 as third digit from right

Creating Regular Expressions

For all strings of 0's and 1's that...

- a) Begin in 1
 - $1(0|1)^*$
- b) End in 1
 - $(0|1)^*1$
- c) Contains 00
 - $(0|1)^*00(0|1)^*$
- d) Do not contain 00
 - $(01|1)^*(\epsilon|0)$

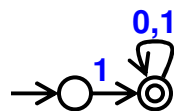
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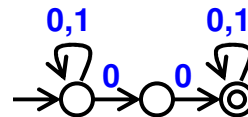
Creating NFA

For all strings of 0's and 1's that...

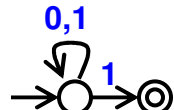
- a) Begin in 1



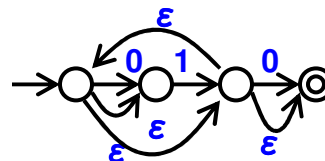
- c) Contains 00



- b) End in 1



- d) Do not contain 00



Based on regular expression

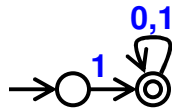
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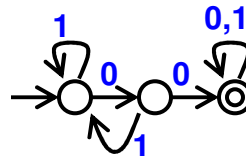
Creating DFA

For all strings of 0's and 1's that...

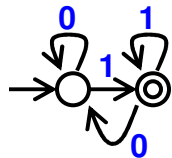
a) Begin in 1



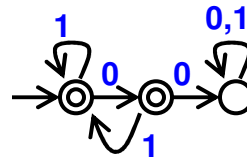
c) Contains 00



b) End in 1



d) Do not contain 00



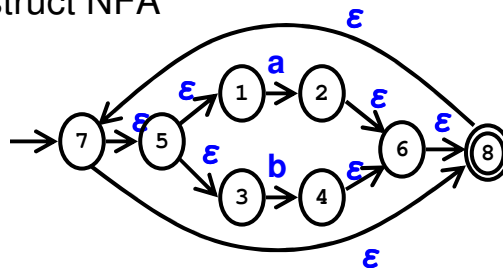
Swap final / non-final states!

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For RE $(a | b)^*$

a) Construct NFA



b) Accept ababbab

7,5,1,2,6,8,7,5,3,4,6,8,7,5,1,2,6,8,7,5,3,4,6,8,7,5,3,4,6,
8,7,5,1,2,6,8,7,5,3,4,6,8 accept

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For RE $(a \mid b)^*$

c) Reduce NFA to DFA

- Start = ϵ -closure(7) = {7,5,1,3,8}
- $R = \{\{7,5,1,3,8\}\}$
- $r \in R = \{7,5,1,3,8\}$ // mark DFA state
- move ({7,5,1,3,8}, a) = {2}
 - $e = \epsilon$ -closure({2}) = {2,6,8,7,5,1,3} // new DFA state
 - $R = R \cup \{2,6,8,7,5,1,3\}$ // add to R
 - $\delta = \delta \cup (\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\})$
- move ({7,5,1,3,8}, b) = {4}
 - $e = \epsilon$ -closure({4}) = {4,6,8,7,5,1,3} // new DFA state
 - $R = R \cup \{4,6,8,7,5,1,3\}$ // add to R
 - $\delta = \delta \cup (\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\})$

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For RE $(a \mid b)^*$

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r \in R = \{2,6,8,7,5,1,3\}$ // mark DFA state
- move ({2,6,8,7,5,1,3}, a) = {2}
 - $e = \epsilon$ -closure({2}) = {2,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})$
- move ({2,6,8,7,5,1,3}, b) = {4}
 - $e = \epsilon$ -closure({4}) = {4,6,8,7,5,1,3} // existing DFA state
 - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$

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For RE $(a \mid b)^*$

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r \in R = \{4,6,8,7,5,1,3\}$ // mark DFA state
- $\text{move}(\{4,6,8,7,5,1,3\}, a) = \{2\}$
 - $e = \varepsilon\text{-closure}(\{2\}) = \{2,6,8,7,5,1,3\}$ // existing DFA state
 - $\delta = \delta \cup (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})$
- $\text{move}(\{4,6,8,7,5,1,3\}, b) = \{4\}$
 - $e = \varepsilon\text{-closure}(\{4\}) = \{4,6,8,7,5,1,3\}$ // existing DFA state
 - $\delta = \delta \cup (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$
- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - No more unmarked states to process
- $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - Since $8 \in F_n$

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For RE $(a \mid b)^*$

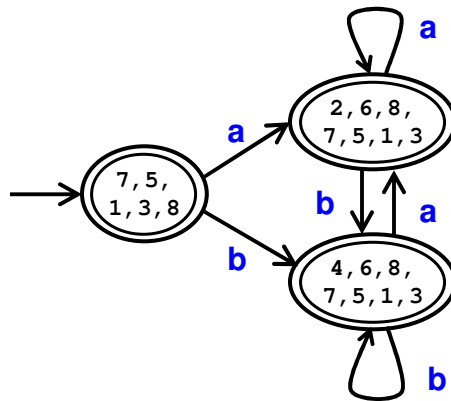
- Resulting DFA
 - $\Sigma = \{a,b\}$
 - $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - $r_0 = \{7,5,1,3,8\}$
 - $F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
 - $\delta = \{$
 - $(\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\}),$
 - $(\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\}),$
 - $(\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}),$
 - $(\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}),$
 - $(\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}),$
 - $(\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$

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For RE $(a | b)^*$

- NFA to DFA reduction pictorial



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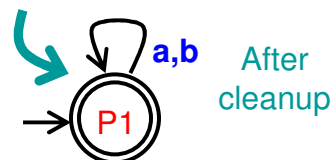
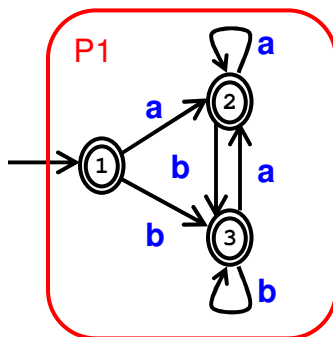
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For RE $(a | b)^*$

d) Minimize DFA

- Initial partitions
 - Accept $\rightarrow \{1, 2, 3\} \rightarrow P1$
 - Reject $\rightarrow \emptyset$
- Split partition?
 - move(1,a) $\rightarrow P1$
 - move(2,a) $\rightarrow P1$
 - move(3,a) $\rightarrow P1$
 - move(1,b) $\rightarrow P1$
 - move(2,b) $\rightarrow P1$
 - move(3,b) $\rightarrow P1$

Not required, minimization done

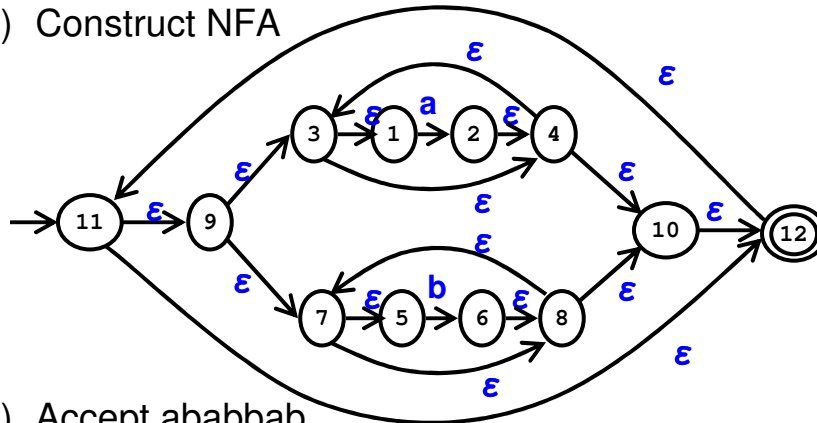


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For RE $(a^* | b^*)^*$

a) Construct NFA



b) Accept ababbab

11,9,3,1,2,4,10,12,11,9,7,5,6,8,10,12,11,9,3,1,2,4,10,12...

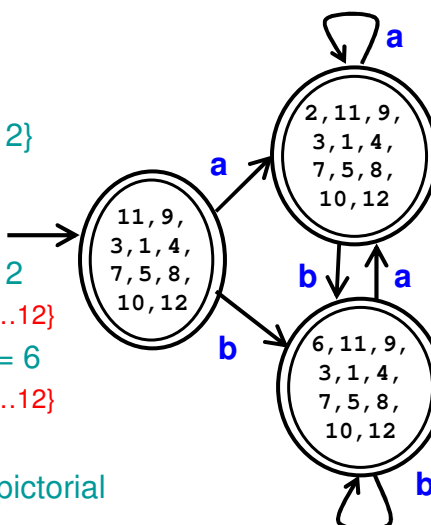
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For RE $(a^* | b^*)^*$

c) Reduce NFA to DFA

- Start = ϵ -closure(1)
- $= \{11, 9, 3, 1, 4, 7, 5, 8, 10, 12\}$
- $R = \{\{11, 9, \dots, 12\}\}$
- $r \in R = \{11, 9, \dots, 12\}$
- move $(\{11, 9, \dots, 12\}, a) = 2$
 - ϵ -closure $(\{2\}) = \{2, 11, 9, \dots, 12\}$
- move $(\{11, 9, \dots, 12\}, b) = 6$
 - ϵ -closure $(\{6\}) = \{6, 11, 9, \dots, 12\}$
- ...
- NFA to DFA reduction pictorial



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For RE $(a^* | b^*)^*$

d) Minimize DFA

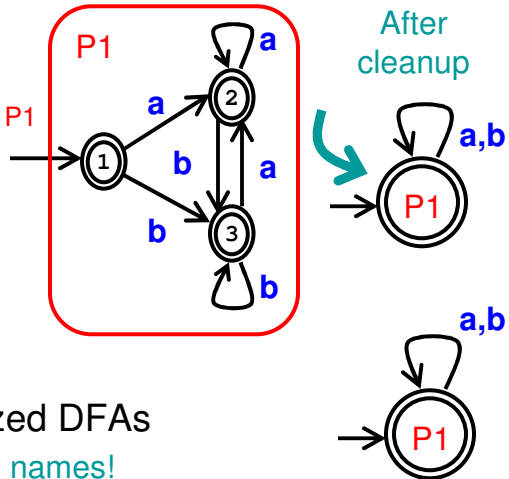
- Initial partitions

- Accept $\rightarrow \{1,2,3\} \rightarrow P1$
- Reject $\rightarrow \emptyset$

- Split partition?

- $\text{move}(1,a) \rightarrow P1$
- ...

Not required,
minimization done



e) Compare 2 minimized DFAs

- Identical up to state names!