1. (10 points) Show that if $7|n^2$ then $7|n$ (without using the Unique Factorization Theorem).
2. (20 points) Show that $\sqrt{7}$ is irrational.
   
   (a) Use the Unique Factorization Theorem.

   (b) Do not use the Unique Factorization Theorem.
3. (20 points) Consider the sequence $w_0, w_1, w_2, \ldots$, where $w_i$ is the number of 1's in the binary representation of $i$.

(a) Write down the first thirteen terms of the sequence (i.e., $w_0, w_1, w_2, \ldots, w_{12}$). No justification needed.

(b) Calculate $\sum_{i=0}^{12} w_i$. No justification needed.

4. (30 points) For each sequence, write a simple, closed-form expression for the $i$th term. HINT: It can sometimes be helpful look at the difference of consecutive terms ($a_{i+1} - a_i$) or the ratio of consecutive terms ($a_{i+1}/a_i$) to see what is happening.

(a) $-\frac{1}{2}, \frac{1}{5}, -\frac{1}{8}, \frac{1}{11}, -\frac{1}{14}, \ldots$,

(b) 3, 5, 11, 29, 83, \ldots,

(c) 2, -3, 6, -13, 28, -59, \ldots,
5. (10 points) Give an explicit mapping to show that the positive, even integers \((\mathbb{Z}^{\text{even, +}})\) have the same cardinality as the integers divisible by 3 \(\{n \text{ such that } 3|n\}\). You should produce a function that is a bijection. It should be an explicit formula not involving cases.

6. (10 points) Find all real numbers \(x\) satisfying
\[
\lfloor x + 1 \rfloor = \lfloor 4x \rfloor.
\]
Show your work. HINT: Every real number \(x\) can be written as \(n + \epsilon\) where \(n\) is an integer and \(\epsilon\) is a real number such that \(0 \leq \epsilon < 1\).