1. (20 points) Let \( P(n) \) be a predicate. Assume you have proven that \( P(9) \) holds and that \( P(n - 2) \rightarrow P(n + 3) \). What do you know? State your answer simply and clearly using modular arithmetic. No justification needed.
2. (20 points) We will use Mathematical Induction to prove that for $n \geq 2$

$$\sum_{j=2}^{n} \frac{1}{(j-1)j} = 1 - \frac{1}{n}$$

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.
3. (20 points) We will use Mathematical Induction to prove that for \( n \geq 1 \)

\[
\sum_{i=1}^{n} i(i!) = (n + 1)! - 1
\]

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.
4. (20 points) We will use Mathematical Induction to show that, for all integers $n \geq 1$, $8 | (3^{2^n} - 1)$.

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.
5. (20 points) We will use Mathematical Induction to show that, for all integers \( n \geq 2 \), if \( x_1, x_2, \ldots, x_n \) are real numbers strictly between 0 and 1 (i.e., \( 0 < x_i < 1 \)), then

\[
(1 - x_1)(1 - x_2) \cdots (1 - x_n) > 1 - x_1 - x_2 - \cdots - x_n
\]

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.