The following exercises are designed to test your understanding of recursion. The functions are defined using a variant of LISP known as meta-LISP. In order to aid your understanding, the function defined in problem 1 is identical to the one below:

\[
drop(x) = \begin{cases} 
  \text{nil} & \text{if } \text{null } x \\
  \text{car } x \text{ cons drop(cdr } x \text{)} & \text{else}
\end{cases}
\]

The idea is that

- \(a \times = \text{car } x\)
- \(d \times = \text{cdr } x\)
- \(n \times = \text{null } x\)
- \(a t x = \text{atom } x\)
- \(a . b = a \text{ cons } b\)
- \(<a> = a \text{ cons } \text{nil } = \text{a list whose single element is } a\)
- \(a * b = \text{concatenate lists } a \text{ and } b \) (i.e. append list \(b\) to list \(a\))
- \(\text{reverse}[x] = \text{reverses the top level list } x. \text{ For example } \text{reverse}[(A B C)] = (C B A). \text{ But } \text{reverse}[((A B C)(D E))] = ((D E)(A B C))\).

1. Consider the function \(\text{drop}\) defined by

\[
\text{drop}[x] \leftarrow \begin{cases} 
  \text{nil} & \text{if } \text{null } x \\
  \text{a x . drop}[\text{d x}] & \text{else}
\end{cases}
\]

Compute (by hand) \(\text{drop}[(A B C)]\). What does drop do to lists in general?

2. What does the function \(\text{r2}\) defined by

\[
\text{r2}[x] \leftarrow \begin{cases} 
  \text{nil} & \text{if } \text{null } x \\
  \text{reverse}[\text{a x}].\text{r2}[\text{d x}] & \text{else}
\end{cases}
\]

do to lists of lists? How about

\[
\text{r3}[x] \leftarrow \begin{cases} 
  \text{reverse}[\text{r4}[x]] & \text{if } \text{null } x \\
  \text{r4}[x] & \text{else}
\end{cases}
\]

\[
\text{r4}[x] \leftarrow \begin{cases} 
  \text{null} & \text{if } \text{null } x \\
  \text{r3}[\text{a x}].\text{r4}[\text{d x}] & \text{else}
\end{cases}
\]

3. Compare the following function with the function \(\text{r3}\) of the preceding example:

\[
\text{r3'}[x] \leftarrow \begin{cases} 
  \text{r3'}[\text{d x}] & \text{if } \text{null } x \\
  \text{r3'}[\text{a x}] & \text{else}
\end{cases}
\]

4. Consider \(\text{r5}\) defined by

\[
\text{r5}[x] \leftarrow \begin{cases} 
  \text{null} & \text{if } \text{null } x \text{ or } \text{null } \text{d x}
  \text{else } [a \text{ r5}[d x]] \text{ cons } \text{r5}[a x . \text{r5}[d \text{r5}[d x]]]
\end{cases}
\]

Compute \(\text{r5}[(A B C D)]\). What does \(\text{r5}\) do in general. Needless to say, this is not a good way of computing this function even though it involves no auxiliary functions.