Graph Implementation

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Graph Implementation

- How do we represent nodes/edges?
  - Adjacency matrix
    - 2D array of neighbors
  - Adjacency list/set/map
    - List/set/map of neighbors
- Important for very large graphs
- Affects efficiency / storage
Adjacency Matrix

- Single **two-dimensional array** for entire graph
- Directed Graph
  - **Unweighted graph**
    - Matrix elements ⇒ boolean
  - **Weighted graph**
    - Matrix elements ⇒ values
  - Let’s see an example of each
- Undirected Graph
  - Let’s see an example for unweighted graph
  - Let’s see an example for weighted graph
- For Undirected graph
  - Only upper / lower triangle matrix needed
  - Since $n_j, n_k$ implies $n_k, n_j$
Adjacency List/Set/Map

- For each node, **store neighbor information in a list, set, or map**
- The main structure can be a list, set, or map

**Directed Graph**
- **Unweighted graph**
  - List or set of neighbors
- **Weighted graph**
  - Each entry keeps track of neighbor and weight
  - Easy to implement with maps
    - Maps of Maps (using HashMaps for efficiency)
  - Let’s see an example of each

**Undirected Graph**
- Let’s see an example for unweighted graph
- Let’s see an example for weighted graph
Additional Examples

• Examples
  • Unweighted graph
    - node 1: \{2, 3\}
    - node 2: \{1, 3, 4\}
    - node 3: \{1, 2, 4, 5\}
    - node 4: \{2, 3, 5\}
    - node 5: \{3, 4, 5\}

• Weighted graph
  - node 1: \{2=3.7, 3=5\}
  - node 2: \{1=3.7, 3=1, 4=10.2\}
  - node 3: \{1=5, 2=1, 4=8, 5=3\}
  - node 4: \{2=10.2, 3=8, 5=1.5\}
  - node 5: \{3=3, 4=1.5, 5=6\}
Graph Properties

- **Graph density**
  - Ratio edges to nodes (dense vs. sparse)
  - For adjacency matrix many empty entries for large, sparse graph

- **Adjacency matrix**
  - Can find individual edge \((a,b)\) quickly
  - Examine entry in array Edge\([a,b]\)
    - Constant time operation

- **Adjacency list / set / map**
  - Can find all edges for node \((a)\) quickly
  - Iterate through collection of edges for \(a\)
    - On average \(E / N\) edges per node
## Complexity

- Average Complexity of operations
  - For graph with $N$ nodes, $E$ edges

<table>
<thead>
<tr>
<th>Operation</th>
<th>Adj Matrix</th>
<th>Adj List</th>
<th>Adj Set/Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Insert edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Delete edge</td>
<td>$O(1)$</td>
<td>$O(E/N)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Enumerate edges for node</td>
<td>$O(N)$</td>
<td>$O(E/N)$</td>
<td>$O(E/N)$</td>
</tr>
</tbody>
</table>
Choosing Graph Implementations

- Factors to consider
  - **Graph density**
  - **Graph algorithm**
    - Neighbor based
      For each node X in graph
      For each neighbor Y of X // adj list faster if sparse
      doWork( )
    - Connection based
      For each node X in ...
      For each node Y in ...
      if (X,Y) is an edge // adj matrix faster if dense
      doWork( )