In Class Exercise – Operational Semantics Derivations
Wed, Sep 30, 2015

These two problems use a subset of the language of arithmetic expressions we saw in lecture:

\[ a ::= n \mid X \mid a + a \]

where \( X \in \text{Var} \) ranges over variables, and a program state \( \sigma : \text{Var} \to n \) maps variables to integers \( n \).

1. Consider the following big-step semantics rules:

\[
\frac{(n, \sigma) \rightarrow n}{(n, \sigma) \rightarrow^{\sigma} n}
\]

\[
\frac{(X, \sigma) \rightarrow \sigma(X)}{(a_1, \sigma) \rightarrow n \quad (a_2, \sigma) \rightarrow m \quad p = n + m}{(a_1 + a_2, \sigma) \rightarrow p}
\]

Write a derivation showing that \((1 + X) + 3, \sigma) \rightarrow^\sigma 6\) if \( \sigma = [X \mapsto 2] \).

2. Consider the following small-step semantics rules:

\[
\frac{a_1 \rightarrow^\sigma a_1'}{X \rightarrow^\sigma \sigma(X)}
\]

\[
\frac{a_1 + a_2 \rightarrow^\sigma a_1' + a_2}{n + a_2 \rightarrow^\sigma n + a_2}
\]

\[
\frac{a_2 \rightarrow^\sigma a_2'}{p = n + m}
\]

Write a sequence of derivations showing that \((1 + X) + 3 \rightarrow^\ast_\sigma 6\) if \( \sigma = [X \mapsto 2] \). Here \( \rightarrow^\ast_\sigma \) is the reflexive, transitive closure of \( \rightarrow_\sigma \).