1. (20pts) **(Image projection)** Consider the cube with points P1, P2, P3, P4, P5, P6, P7, P8 and 3D coordinates in the world coordinate system as given in Figure 1. A calibrated camera with focal length \( f = 1 \) whose origin is at \((0, 0, -3)\) and which has a rotation of \(-45\) degrees around the Y-axis with respect to the world coordinate system takes an image of the cube. The image coordinates of the corners of the cube are labeled \(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\). Remember a rotation of angle \(a\) around the Y-axis can be expressed by the 3X3 rotation matrix 
\[
R = \begin{pmatrix}
\cos a & 0 & \sin a \\
0 & 1 & 0 \\
-\sin a & 0 & \cos a
\end{pmatrix}
\]
(a) Derive the projection matrix mapping homogeneous world coordinates to homogeneous image coordinates.
(b) Compute the homogeneous and the non-homogenous image coordinates of points \(p_5, p_6, p_7, p_8\).
(c) Derive the non-homogenous coordinates of the 3 vanishing points corresponding to the 3 parallel lines.
(d) Compute the vanishing point of the line \(P_5 P_7\)
(e) How would the camera need to be positioned with respect to the Cube such that 2 of the vanishing points are ideal (that is are at infinity)?

![Figure 1](image-url)
2. (10pts) If $P$ is a (3X4) camera projection matrix and $C$ is the camera center (in projective coordinates), then show that $PC=0$.

3. (10pts) In a 3X4 camera matrix, what is the geometric meaning of the 4 column vectors?

4. (30 pts) (Calibration) Suppose we are given a 3X4 camera matrix $P$:

$$
\begin{bmatrix}
3.53e+2 & 3.39e+2 & 2.77e+2 & 1.44e+6 \\
-1.03e+2 & 2.33e+1 & 4.59e+2 & -6.32e+5 \\
7.07e-1 & -3.53e-1 & 6.12e-1 & -9.18e+2
\end{bmatrix}
$$

Compute the camera center and the calibration parameters.

5. (15 pts) (Cross ratio invariance) Consider 3 points $a,b,c$ lying on a straight line in an image. They are images of known points $A,B$ and $C$ lying on a line in 3D. Show how to compute the vanishing point of line $ABC$ from the ratio: $ab/bc$ that can be measured in the image. (*Hint: Use the cross-ratio invariance*)

6. (20 pts) (Projective reconstruction using coplanar points). Assume two sets of four known coplanar points in the scene $\{A,B,C,D\}$ and $\{E,F,G,H\}$ giving images $\{a,b,c,d\}$ and $\{e,f,g,h\}$ for an non calibrated camera with camera center $O$. Consider an unknown point $M$ in the scene producing an image $m$.

(a) Show how to determine the viewing ray $Om$.

(b) Use (a) to develop a method for finding the camera center.

7. (90 pts) A robot vehicle is equipped with a camera. The camera image is a rectangle of height 500 pixels and width 600 pixels. The focal length is 690 pixels. The center of projection $C$ of the camera is 3 m above the ground. The optical axis of the camera makes a 20 degree angle with the ground, so that the camera looks slightly downward toward the road. Large squares of size 4 m have been painted everywhere along the median lines of roads to facilitate automatic vehicle navigation. Two of the sides of the squares are parallel to the road edges. The vehicle is on a flat road.
where it sees only two squares. The vehicle faces the first square such that the optical axis of the camera passes through the center of the square, and is perpendicular to two sides of the square. Because the road turns, the next square is at an angle with the first square. The centers of the squares are on a circle of radius 100 meters (which is also the radius of the road turn), and the arc between the two centers is 10 degrees.

a. Compute the pixel positions of the images of the corners of the two squares.
b. Make a drawing of the images of the two squares and of the rectangular frame of the image using Matlab.
c. To navigate, the robot computes a projective transformation (in other words, a projectivity, or homography) that transforms the images of the squares back to their actual geometry on the road, in order to compute how the road turns. To do that, it does not use the camera focal length and the camera position with respect to the road. Instead, it uses its knowledge that the squares it sees are always of size 4 m, and computes a projectivity matrix using the four corners of the closest square it sees. Using the images of the square corners found in (a), compute this matrix using Matlab.
d. Draw the two reconstructed squares. Compute the radius of the road turn.

8. (5 pts) Is this parachuter higher or lower than the person taking this picture?