

Practice Problems – Operational Semantics Sample Solution

1.

$$\frac{\langle c_0, \sigma \rangle \longrightarrow \sigma' \quad \langle b, \sigma' \rangle \longrightarrow \text{false}}{\langle \text{for}(c_0; b; c_1) c_2, \sigma \rangle \longrightarrow \sigma'} \quad \frac{\langle c_2, \sigma_0 \rangle \longrightarrow \sigma_2 \quad \frac{\langle c_0, \sigma \rangle \longrightarrow \sigma_0 \quad \langle b, \sigma_0 \rangle \longrightarrow \text{true}}{\langle c_1, \sigma_2 \rangle \longrightarrow \sigma_3} \quad \langle \text{for}(\text{skip}; b; c_1) c_2, \sigma_3 \rangle \longrightarrow \sigma'}{\langle \text{for}(c_0; b; c_1) c_2, \sigma \rangle \longrightarrow \sigma'}$$

2. (a) $\frac{}{(\lambda x.42) 13 \rightarrow 42}$

(b) $\frac{\overline{(\lambda x.x) (\lambda y.y) \rightarrow \lambda y.y}}{\overline{((\lambda x.x) (\lambda y.y)) (\lambda z.z) \rightarrow (\lambda y.y) (\lambda z.z)}}$

(c) $\frac{\overline{(\lambda x.\lambda y.x) 42 \rightarrow \lambda y.42}}{(\lambda x.x) ((\lambda x.\lambda y.x) 42) \rightarrow (\lambda x.x) (\lambda y.42)}$

3.

$$\frac{\text{BETA}}{(\lambda x.e) \rightarrow (\lambda x.e)} \quad \frac{\text{INT}}{n \rightarrow n} \quad \frac{\text{APP}}{e_1 \rightarrow (\lambda x.e) \quad e_2 \rightarrow v \quad e[x \mapsto v] \rightarrow v'}{e_1 e_2 \rightarrow v'}$$

$$\frac{\lambda x.x \rightarrow \lambda x.x}{\frac{\frac{\frac{\lambda x.\lambda y.x \rightarrow (\lambda x.\lambda y.x)}{42 \rightarrow 42} \quad (\lambda y.x)[x \mapsto 42] \rightarrow (\lambda y.42)}{(\lambda x.\lambda y.x) 42 \rightarrow \lambda y.42} \quad (\lambda x.x)((\lambda x.\lambda y.x) 42) \rightarrow (\lambda y.42)}{x[x \mapsto (\lambda y.42)] \rightarrow (\lambda y.42)}}$$

2 4.