Practice Problems – Operational Semantics

1. Recall the language IMP from class:

\[
\begin{align*}
a & ::= n \mid X \mid a_0 + a_1 \mid a_0 - a_1 \mid a_0 \ast a_1 \\
b & ::= bv \mid a_0 = a_1 \mid a_0 \leq a_1 \mid \neg b \mid b_0 \land b_1 \mid b_0 \lor b_1 \\
c & ::= \text{skip} \mid X := a \mid \text{if } b \text{ then } c_0 \text{ else } c_1 \mid \text{while } b \text{ do } c \\
bv & ::= \text{true} \mid \text{false}
\end{align*}
\]

Suppose we extend the language with a C-style for loop:

\[
c ::= \ldots \mid \text{for}(c_0; b; c_1) \; c_2
\]

Write down big-step operational semantics for “for.” You may not use “while” in the hypotheses of your “for” rules. Hint: the “skip” command may come in handy.

2. Here is the lambda calculus, extended with integers, and its semantics:

\[
\begin{align*}
e & ::= v \mid x \mid e \; e \\
v & ::= n \mid \lambda x.e
\end{align*}
\]

\[
\begin{align*}
\text{Beta} & : (\lambda x.e_1)v_2 \rightarrow e_1[x \mapsto v_2] \\
\text{Left} & : e_1 \rightarrow e'_1 \\
\text{Right} & : e_2 \rightarrow e'_2
\end{align*}
\]

Draw derivations showing that the following reductions hold:

(a) \((\lambda x.42)\) 13 \rightarrow 42

(b) \(((\lambda x.\lambda y.y))(\lambda z.z)\) \rightarrow \((\lambda y.y)(\lambda z.z)\)

(c) \((\lambda x.\lambda y.x 42)\) \rightarrow \((\lambda x.x)(\lambda y.42)\)

3. Write down big-step semantics for lambda calculus that are equivalent to the rules above (for terminating programs).

4. Draw a derivation of the following in your big-step semantics: \((\lambda x.x)(((\lambda x.\lambda y.x) 42)\rightarrow (\lambda y.42)\)