

# Practice Problems – Type Systems

## Sample Solution

1. (a)  $\frac{}{\cdot \vdash 42 : int}$
  - (b)  $\frac{x : int, y : int \vdash y : int}{y : int \vdash \lambda x : int. y : int \rightarrow int}$
  - (c)  $\frac{\frac{y : int, x : int \vdash x : int}{x : int \vdash \lambda y : int. x : int \rightarrow int}}{\cdot \vdash \lambda x : int. \lambda y : int. x : int \rightarrow int \rightarrow int}$
  - (d) Let  $A = + : int \rightarrow int \rightarrow int$ . See Figure 1.
2. There are many possible examples, e.g.,  $(\lambda x : int \rightarrow int. 42)$   $(\lambda y : int. 1 \ 2)$ .
3. (a) (as shown in problem set)

(b)

$$\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha}{x : \alpha \vdash \lambda y. x : \beta \rightarrow \alpha}}{\cdot \vdash \lambda x. \lambda y. x : \alpha \rightarrow \beta \rightarrow \alpha}$$

Solution:  $\alpha$  and  $\beta$  are unconstrained  
 Type :  $\alpha \rightarrow \beta \rightarrow \alpha$ .

(c)

$$\frac{\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha \quad y : \beta, x : \alpha \vdash y : \beta \quad \alpha = (\beta \rightarrow \gamma)}{y : \beta, x : \alpha \vdash x \ y : \gamma}}{x : \alpha \vdash \lambda y. x \ y : \beta \rightarrow \gamma}}{\cdot \vdash \lambda x. \lambda y. x \ y : \alpha \rightarrow \beta \rightarrow \gamma}$$

Solution:  $\alpha = (\beta \rightarrow \gamma)$ , and  $\beta$  and  $\gamma$  are unconstrained.  
 Type:  $(\beta \rightarrow \gamma) \rightarrow \beta \rightarrow \gamma$ .

(d)

$$\frac{\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha}{x : \alpha \vdash \lambda y. x : \beta \rightarrow \alpha}}{\cdot \vdash (\lambda x. \lambda y. x) : \alpha \rightarrow \beta \rightarrow \alpha} \quad \cdot \vdash 3 : int \quad (\alpha \rightarrow \beta \rightarrow \alpha) = (int \rightarrow \gamma)}{\frac{\cdot \vdash (\lambda x. \lambda y. x) \ 3 : \gamma}{\cdot \vdash 42 : int \quad \gamma = (int \rightarrow \delta)}}{\cdot \vdash (\lambda x. \lambda y. x) \ 3 \ 42 : \delta}$$

Solution:  $\alpha = \beta = \delta = int$ ,  $\gamma = int \rightarrow int$ .  
 Type:  $int$ .

$$\begin{array}{c}
\frac{f : \text{int} \rightarrow \text{int}, A \vdash f : \text{int} \rightarrow \text{int}}{f : \text{int} \rightarrow \text{int}, A \vdash f : \text{int} \rightarrow \text{int}} \quad \frac{f : \text{int} \rightarrow \text{int}, A \vdash 42 : \text{int}}{f : \text{int} \rightarrow \text{int}, A \vdash f \ 42 : \text{int}} \\
\hline
A \vdash (\lambda f.f \ 42) : (\text{int} \rightarrow \text{int}) \rightarrow \text{int} \\
\hline
A \vdash (\lambda f.f \ 42) (\lambda x.x + x \ 3) : \text{int}
\end{array}$$

$$\begin{array}{c}
\frac{x : \text{int}, A \vdash + : \text{int} \rightarrow \text{int} \rightarrow \text{int}}{x : \text{int}, A \vdash + x : \text{int} \rightarrow \text{int}} \quad \frac{x : \text{int}, A \vdash 3 : \text{int}}{x : \text{int}, A \vdash 3 : \text{int}} \\
\hline
x : \text{int}, A \vdash + x \ 3 : \text{int} \\
\hline
A \vdash (\lambda x.x + x \ 3) : \text{int} \rightarrow \text{int}
\end{array}$$

Figure 1: Derivation for 2.a.iv. The types of function arguments have been omitted for space reasons.