Overview of Lectures 15 and 16: Security Games

The main topic of these lectures is about fair allocation mechanisms. In these mechanisms both the fairness and the efficiency are considered. The first section is about allocation of divisible resources, the second section is about allocation of indivisible resources, the third section is about allocating of food to food banks, and two final parts are about course match.

Details

1. Allocation of divisible resources without money

We assume some sort of structure on the users’ demand which is called Leontief preferences, and states that the utility of an agent is the fraction of its dominant resource that it can actually use, given its proportional demands and its allocation of the various resources. For example, an agent that requires twice as much CPU as RAM to run a task prefers to be allocated 4 CPU units and 2 RAM units to 2 CPU units and 1 RAM unit, but is indifferent between the former allocation and 5 CPU units and 2 RAM units. Let $w_i$ denote the units of resource $i$ that an agent needs to run a task, and let $x_i$ denote the units of resource $i$ that is allocated to this agent. The utility of the agent in this allocation is as follows:

$$u(x_1, \ldots, x_m) = \min\{\frac{x_1}{w_1}, \ldots, \frac{x_m}{w_m}\}$$

One form of fair allocation mechanism in this model is called Dominant resource fairness (DRF) which is about equalizing the largest shares or dominant shares of the agents in the system. This mechanism is proposed by Ghodsi et al. [4] for static setting, and it assumes all agents are present from the beginning and all the job information is known upfront. Kash et al. [5] then relaxed this to dynamic setting. In their model, Agents are arriving over time and Job information of an agent only revealed upon arrival. However, they have this assumption that the total number of agents and all the resources are known in advance.

First dynamic model

There are $M$ agents and $K$ resources the number of agents and resource information are known in advance, but agents are arriving in different time steps and their demands which are proportional reveals at arrival. In addition each agent requires every resource and any agent has a dominant resource. When agents arrive they can not depart the system. Moreover if we can no decrease the amount of allocated resources to an agent.

We first focus on a very simple dynamic mechanism allocation. At every step $i$ that agent $i$ arrives, we allocate some amount of each resource over the $i$ present agents, and we terminate after the final agent arrives. There are some known properties for the static setting that we are interested to define in this dynamic setting. The detailed definition of these properties are in Figure [1]. There is an impossibility result which states there is no mechanism that guarantees both Envy freeness and Pareto optimality in the dynamic setting. The intuition is that DPO is going to require you to give too much to too many people, and once you allocate too much resource and you can not revoke that, there can be some arriving agents that envy existing agents. So, we want a mechanism that cares about both these properties since if we drop one of them there is a trivial mechanism that guarantees the other one. For example, if we drop DPO, a trivial mechanism is to not allocate anything to
anyone until the end. So, we relax these definitions to something that sounds good and is also possible. The relaxed version of DEF is as follows: If agent $i$ envies agent $j$, then $j$ must have arrived before $i$ did, and must not have been allocated any resources since $i$ arrived. There is also another form of envy freeness in the dynamic setting which is called forward envy freeness. In this form, an agent may only envy agents that have arrived after her. However this is strictly weaker than backward envy freeness. A trivial forward envy free mechanism is to allocate everything to the agent who comes first. So, the mechanism that satisfies DPO and our DEF is as follows:

1. Agent $i$ arrives
2. Start with (previous) allocation of step $i-1$
3. Keep allocating to all agents having the minimum dominant (largest) share at the same rate until a $\frac{i}{M}$ fraction of at least one resource is allocated

This mechanism is type of water filling algorithm that satisfies DPO the mentioned DEF, Sharing incentives and Strategyproofness. However, in this mechanism it is always better to be the first one rather than the last one.

Sometime we really want fairness and we do not want to chase efficiency. One way to do this is wait until the last agent arrives the do a static envy free and Pareto optimal allocation. But the question is that can we allocate more resources early? There is the Cautious Dynamic Pareto Optimality (CDPO) which states at every step, allocate as much as possible while ensuring EF can be achieved in the end irrespective of the future demands. The mechanism based on this definition is called CautiousLP and it is going to satisfy strong envy freeness. However this mechanism is too inefficient since this constraint forces you to not allocate many resources to any agent until the very end.

In Figure 1 we have some nice experimental results that are based on real data. We compare DynamicDRF and CautiousLP with provable lower and upper bounds. (The data is from Google). There are two objectives Maxsum (Maximize the sum of dominant shares) and Maxmin (Maximize the minimum dominant share). With

<table>
<thead>
<tr>
<th>Property</th>
<th>Static (DRF)</th>
<th>Dynamic (Desired)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Envy freeness</td>
<td>EF: No swaps.</td>
<td>EF: No swaps at any step.</td>
</tr>
<tr>
<td>Sharing incentives</td>
<td>SI: At least as good as equal split.</td>
<td>SI: At least as good as equal split to every present agent at all steps.</td>
</tr>
<tr>
<td>Strategyproofness</td>
<td>SP: No gains by misreporting.</td>
<td>SP: No gains at any step by misreporting.</td>
</tr>
<tr>
<td>Pareto optimality</td>
<td>PO : No “better” allocation.</td>
<td>DPO: At any step k, no “better” allocation using $k/n$ share of each resource.</td>
</tr>
</tbody>
</table>
Maxsum objective, these two mechanisms are not doing very well in terms of efficiency but they are doing better than the lower bound. However, when we talk about Maxmin objective the mechanisms start doing better.

## 2 Allocation of indivisible items

In the items are indivisible it is not always possible to have an envy freeness allocation even in the static setting \(^1\). For example, if we have only one item and two agents that both value the item equally. So, we need to change our desiderata from previous part. The new model is as follows. There are \(K\) agents, each with private utility for each of \(M\) items. Items arrive one at a time, agents bid “like” or “dislike” on items when they arrive and the mechanism assigns items randomly to one of the agents who liked the item. This mechanism is Strategyproof, and the strategy for agents is to like any item that has nonzero utility for them, but it has some bad properties, for example, even if an agent likes all the items, there is still a chance that the mechanism allocates no item to her. To fix this we change the mechanism such that in each round it allocates the item randomly amongst Likers that have received the fewest overall number of items. This new mechanism that is called balanced-like mechanism guarantees that any agent receives at least one item per every \(K\) she likes, but it is not Strategyproof in general. Balanced-like mechanism is Strategyproof only when we have two agents and 0/1 utilities.

### Social welfare and Fairness:

So, the system can be gamed in general case, but what does this gaming mean for social welfare or fairness of the system? Authors of this paper were motivated by working with a food bank in Australia, where they expected to work with relatively unsophisticated dispatchers who bid on food. In cases like this we care about both objectives. We care about social welfare because we do not want to starve a single food bank over and over again. So, in general bidding strategically is very bad for social welfare, and we measure that by comparing sincere behavior against a worst case result from a set of Nash profiles. There is extremely bad result in this paper that states there are instances with 0/1 utilities and \(K\) agents where: the egalitarian, utilitarian welfare with sincere play under like, balanced-like is \(K\) times the corresponding welfare under a Nash profile.

### Envy freeness:

Two types of envy freeness are defined for this model.
1. Ex-ante envy freeness: Before the final allocation, Does any agent expect envies or not?

2. Ex-post envy freeness: After the items are allocated is an agent envies?

Like mechanism is ex-ante envy free since each items allocation is independent of past allocations. Assume first m-1 allocations are EF. There is a simple proof by induction. Item m arrives. Each of j \( \geq k \) agents with utility 1 receives item in \( 1/j \) of possible worlds. So the mechanism is still EF. However this mechanism is not ex-post EF since it is possible that this mechanism assigns all the items to one agent which cause unbounded envy. Using similar arguments, paper shows that balanced-like mechanism under 0/1 utilities is Ex-ante envy free and bounded ex-post envy free(with at most 1 unit of envy).

3 Allocation of food to food banks

We have hungry people out there in the whole nation that get their meals from churches, soup kitchens and etc [6]. These places get their foods from regional food banks. These foods exactly come from large food manufacturers, distributors and also smaller grocery stores that give these foods as a donation to food banks. So, this system needs a large organization which is called Feeding America. This organization has ties with food banks and with the donors. Many donors give the food directly to Feeding America, who then allocates to food banks this type of donation is called yellow pounds. There is also another type called blue pound that is from a particular donor to a particular food bank. So, Feeding America needs to ensure that food ends up with whose need is greatest taking into account taking transportation costs, spoilage and storage issues into account.

**Challenges** Note that, the difficulty is not in estimating a measure of aggregate need in a service area. Data on population residing, income levels and usage of food pantries and soup kitchen can reflect reasonably well aggregate food needs. However the some challenges are as follows:

1. Food Banks receive 20% food from Feeding America. Feeding America knows nothing about rest 80%
2. There are some transitory variation, for example, if a food bank got eggs from local produce, wont need eggs FA is offering that week.
3. “Food richness” (food banks having close ties with local manufacturers/distributors)
4. Regional Diets vary, so demands are different.
5. Some foods are more valuable than others to food banks, for example, frozen meat is more valuable than potato chips.
6. How to trade-off quantity vs. quality in allocations? Some food banks need to feed lots of people so they prefer quantity over quality.

So, knowing these challenges how Feeding America allocates foods? We first introduce the Feeding Americas pre-2005 setup. In this mechanism they have a list of food banks. For any new donation FA calls the next food bank in the list. Food bank can either accept or deny donation. If they accept they are liable for transportation costs. Fresh produce is typically offered to the nearest food bank to the donor because of spoilage issues. Which food bank is next in the list for new load is decided by a metric of need called goal factor, which is a weighted measure of relative poverty of food banks service area compared to the nation and the relative population of food banks service area. Once they know the goal factor they calculate goal pounds is the total number of pounds of food that a food bank should receive until the end of the year. Food banks are ranked on goal pounds relative to pounds received (i.e. a food bank who is furthest below its goal pounds will be ranked highest), and
food is offered to one with the highest rank. Pound received is the pound of the food that is offered not the food that is delivered, which means if FA calls a food bank, either they accept or not their rank goes down. (The main reason they may reject is the transportation costs.) Some drawbacks of this system is as follows:

1. Some food banks always got undesired load.

2. No demands at food bank side indicators involved in allocation process.

3. Assignment system treats all food equally.

4. Food spoils and goes to trash since some small food banks does not have enough storage facilities and when they accept a truck load they have to trough out some of it.

5. Feeding America turns down donations when it feels it cannot place them, which results less food for hungry people.

In 2004, 14 people including Canice Prendergast came together to evaluate and improve allocation mechanism. There were four academics from University of Chicago, three from senior staff at Feeding America and rest were directors of member food banks. The main goal was to incorporate unknown food bank preferences and constraints. The result was pricing for foods or ask food banks to bid on food. This method will reveal their unknown preferences, but we need to give them some sort of budget since otherwise they will bid on everything. In addition in the case of real money for the budget we can not make sure that the neediest one has the highest budget because people can go for found raising and get more money from some other sources. So, because of mentioned problems they come up with something called funny money or shares. This new system is called choice system.

Shares can be used to bid only on yellow pounds and they can not be traded for real money or for anything other than auction market. Shares are initially allocated to food banks based on its goal factors. (Neediest receives highest budget.) There is a website where feeding America posts all the donations offers it has received with details like pickup location, pickup date, etc. Then food banks uses its shares to bid on any offering lot that they wish and could afford. Finally the winner of the auction is the highest bidder and the price of highest bid get subtracted from their budget. In addition, any items that did not sell would be carried over next day for more bidding, and all shares that were spent on a given day are re-distributed at midnight. Moreover, food banks can bid jointly on items by choosing fractional bids and pay accordingly if they win. This helps small food banks when they cannot take whole offered share. Also, food banks can delegate bidding to Feeding America by explaining their needs, so feeding America bids on their behalf. This helps small food banks who may not have infrastructure to have computers or internet connection.

Hard to move product and negative prices: FA allows negative prices for goods from second day, if they did not sell on first day. On the first day of offering, lowest bid possible is 0, but from the second day, good is assigned to the food bank that offers the smallest number of negative shares. On winning such bid, food bank gets bonus shares, where shares gets credited for buying that hard to move product. Offering bonus shares creates win-win situation for both. Feeding America can maintain donor relations, and food banks earn shares for buying food. Again, this feature of not giving negative shares on first day was offered to help small food banks. If they did not check offers everyday, they might miss an opportunity to get bonus shares.

Maroon pounds: Maroon Pounds are foods that are added on auction market by food banks. This happens when a food bank already has some food, perhaps from other source for which it may not be the highest value user, or sometimes, food banks already has something, but wants something else. Ability to resell food may make a food bank accept donation when they cannot use themselves, but someone else can Maroon pounds are
not eligible for bonus shares and they are taxed - 10%. What it implied was, if a food bank is putting food onto market, they probably had more than enough for themselves. Imposing tax in some way was leveling food rich food banks budget.

Outcomes All food banks quickly engaged in the bidding process. Within first 7 months, 97% food banks won at least one load. No food bank chose to delegate bidding to Feeding America except on temporary basis (e.g. director is out of town). In addition this system is transparent and everyone has fair chance to bid on anything they like. Also, There is general acceptance now that smaller food banks have done especially well from the. Moreover, pounds of food on the system rose by 50 million in first seven months and from 2006 to 2012, 15 million on avg. maroon pounds were sold.

There arent any robust methods to estimate merits of choice system. But, staff at Feeding America and all member food banks speak very well of the new system. That in itself, speaks a lot.

4 Combinatorial Assignment Problems and Course Match

There are some sort of problem with the maxmin fairness mechanism for resource allocation. For example there are some cases that the maxmin fairness is very bad for efficiency. So we introduce a new mechanism that tries to be fair and also a little more efficient. This mechanism uses a market with funny money and it is called competitive equilibrium from equal incomes. This mechanism gives the agents an equal share of funny money, agents report their preferences over sets of items and computer finds prices such that when each agent chooses its most favored set that it can afford, the market clears. Then it assign all resources to agents based on their demands and these computed prices. AN example of allocating divisible resources with this mechanism is as follows:

We have a cake and a doughnut and two agents A and B with 1 of funny money. Agent A values cake $\frac{1}{2}$ of the doughnut while agent B values cake $\frac{1}{4}$ of the doughnut. The market clearing prices are: cake = $2.5$, doughnut = $8.5$ since half a cake and half a doughnut maximizes the utility of both agents.

This CEEI is envy-free since given the prices, you bought the best bundle you could afford and if you envy somebody elses bundle, you couldve purchased it. Also, it is Pareto-efficient because finally the market is cleared a Pareto step involves taking a resource from one agent and giving it to somebody new, but this lowers their utility. However, it is not strategy proof. The intuition for this is that CEEI clears the market, which lets agents to game the system by requesting more underutilized resources.

DRF VS CEEI Assume we have two resources CPU and RAM, and we have two agents. $A_1$ needs 1 unit of CPU for any 4 units of RAM while $A_2$ needs 3 units of CPU for any one unit of RAM. The allocations are illustrated in Figure 4. If $A_2$ make a fake claim by saying that she needs 3 units of CPU for any 2 units of RAM. Figure 4 shows that after this claim CEEI changes the allocation of both resources in favor of agent $A_2$. So, DRF is more fair, and CEEI has better utilization.

CEEI for indivisible items: In this case, it is possible that market clearing prices even does not exist, but we try to give approximate-CEEI. The main idea is to give agents slightly different, but roughly equal budgets. For each agent, we draw budget from $[1, 1 + B)$, while $0 < B < \min(1/m, 1/(k – 1))$ and $k$ is capacity of the agent. Here, if we set $B = 0$, this is a simple CEEI. This still feels fair since $B$ is very small and we are drawing budgets uniformly at random so no one getting favor of this. approximate-CEEI always exist if $B > 0$ and budgets are unequal. In addition the market approximately clears. There exist prices that clear the market to within an error of at most $\sqrt{k} \times \frac{m}{2}$. This error does not depend on the number of agents and it goes to zero as a fraction of the underlying endowment.
5 Course Match

There are bunch of students and bunch of courses. What we want to do is to allocate bundles of courses to students. We are subject to various different constraints that are as follows:

1. Each student registers for more than one course.
2. Heterogeneous preferences exist. Each student has different interests and wants different type of courses.
3. Courses meet at the same time, so the bundle of courses that one student can have is limited.
4. Courses have limited capacity. Each course has a limited number of seats.

the focus of this paper [3] is on efficiency and fairness in the sense that we want to make it approximately envy free. Since it is course matching we can not use real money. This problem’s focus is more on Wharton School of Business. They initially used an auction based mechanism. In this mechanism, they gave students some funny money and let them bid on courses over multiple rounds. However they have some incentive problems that do not exist with real money which decreases students satisfaction.After that, Burdish offered a model[2] that gives us efficiency, fairness and incentives, however it has three major issues that are as follows:

1. They could violate capacity constraints
2. It is hard to implement
3. It assumes that students can report their preferences accurately.

To solve the issues that the previous method had, Burdish et al. [3] offered a new model that we will discuss now. In this problem, there are $M$ courses and $N$ students. Also, $q_j$ and $\hat{q}_j$ respectively denote The target capacity and the maximum capacity of course $j$. In addition $\Psi_i$ denotes the set of permissible course bundles that student $i$ can take ($\Psi_i \subseteq 2^M$), and $B_i$ denotes the budget that is assigned to this student, which are approximately equal. The goal here is to find the market clearing prices for the courses, such that if a student
gets a bundle that they can afford, given their specific budget, the market approximately clears. This can be represented as a mixed integer program. (see Figure $5$)

In the previous mechanism we had capacity issues, so here we define $z_j$ as the clearing error that is the number of students subscribed in course $j$ minus $q_j$ (the target capacity of course $j$). There are two types of error that we can have here, undersubscription and oversubscription. So, in the new mechanism we also may violate capacity constraints.(see Figure $5$) We define the overall clearing error as $\alpha = \sqrt{\sum_j z_j^2}$, and we want it to be as low as possible. There is this upper-bound for $\alpha$ that states there is a price vector whose error is bounded by $\sqrt{\frac{KM}{2}}$, where $k$ is the maximum number of courses that one student can take.

In addition in this mechanism they want it to be simple for the students to report their preferences, so they need a way easier than just reporting the complete list. One of the things they focus on this paper is giving the students the capability of reporting their preferences correctly and easily. One of the things that students can assign a utility from 1 to 100 to each course which shows their preferences. They also let students to assign pairwise utilities to courses that they want.

The most interesting part is the calculation part, which is in three stages. The first stage, is the price vector search, the second is eliminating over-subscription, and the reducing under-subscription.

The heuristic search is performed over the price space and is composed of a series of search starts until the allotted time for searching is reached. It begins with a randomly generated price vector $V$, each neighbor is a permutation of $V$. If a neighbor yields a course allocation identical to one of the previous vectors, drop it. The remaining neighbor with the lowest clearing error based on the target capacities $q_j$ is selected as the new candidate vector $V'$ (even if the error of $V' > V$). This stage requires a lot of computational power. To reduce the time to compute them, we can parallelize computation.

Clearing error may appear either as over-subscription or under-subscription, but Eliminating over-subscription error is essential part of making this approach practical. The Eliminating stage relies on the fact that demand for any single course $j$ is monotonically decreasing in course $j$’s price. So, it iteratively identifies the most over-subscribed course and raises its price. Since prices only increase it Converges to a solution with no over-subscription.
After Stage 2, the solution is feasible, but now we however have empty seats. It is likely that they acquired quickly in the drop/add period but we do not want that. Since preference matters for us and Drop/Add period rewards the early birds, so it may create some issues with actual and perceived fairness. So, what they do is this stage is that they increase the budget of everyone by 10%, and then they solve it again with the updated budgets. What any student possibly get in this stage is the courses that was assigned to them in the previous stage plus some open seats. Since budgets only increase, allocation can only improve. The goal here is that students with strongest preferences should be assigned the empty seats. You may think that after this stage students may receive schedules that are even better than they could afford, but this is not seen as an issue for various reasons. essentially still with this mechanism students selected early have an advantage, since many open seats are available for them. What they do is to order second year students before first years (Their program is only 2 years), and within a year, they order by tie-breaking budget. So, people with smallest budget will act first.

The fairness and efficiency of this mechanism are open questions. They also described some failed alternatives. One of them is to set the price of courses to zero, then give everyone what they want and just drop Students from over-subscribed courses. The problem with this mechanism is that students no longer get the best they can afford. The second one is artificially lower target capacities, then if you had over-subscription then adjust that capacity.

References


