THIS CLASS:

COMBINATORIAL ASSIGNMENT PROBLEMS & COURSE MATCH

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Thanks to: John Kubiatowicz (JK)
RECALL: DRF

Proportional demands (a.k.a. Leontief preferences)

\[ u(x_1, \ldots, x_m) = \min \left\{ \frac{x_1}{w_1}, \ldots, \frac{x_m}{w_m} \right\} \]

**Dominant resource**: resource the agent has the biggest share of out of all resources available:

- 16 CPUs, 10 GB available, user allocated 4 CPUs, 8 GB
- Dominant resource is GB, because \(4/16 \text{ CPU} < 8/10 \text{ GPU}\)

**Dominant share**: fraction of dominant resource allocated

- Above, dominant share is \(8/10 = 80\%\)

**DRF**: application of max-min fairness to dominant shares

- Equalize the dominant share amongst agents
STATIC DRF MECHANISM
Dominant Resource Fairness = equalize largest shares
(a.k.a. dominant shares)
ALTERNATIVE: MAKE A MARKET

Competitive Equilibrium from Equal Incomes (CEEI):

- Agents report their preferences over sets of items
- Give agents an equal budget of funny money
- Computer finds prices that clear the market
  - That is, prices such that when each agent chooses its most favored set that it can afford, the market clears
- Assign all resources to agents based on their demands and these computed prices
CEEI EXAMPLE: DIVISIBLE RESOURCES

Supply: \{1 \text{ cake}, 1 \text{ doughnut}\}

Two agents, both with $1 (funny money), capacity of 1

- \text{A}: \text{cake} = 1/2, \text{doughnut} = 1
- \text{B}: \text{cake} = 1/4, \text{doughnut} = 1

Market clearing prices: \text{cake} = $2/5, \text{doughnut} = $8/5

- \text{A} wants to max
  \begin{align*}
  &1/2c + 1d \\
  \text{s.t.} &c + d \leq 1 \\
  &p_c c + p_p d \leq 1
  \end{align*}

  Max: \frac{1}{2} \text{ cake}, \frac{1}{2} \text{ doughnut}

- \text{B} wants to max
  \begin{align*}
  &1/4c + 1d \\
  \text{s.t.} &c + d \leq 1 \\
  &p_c c + p_p d \leq 1
  \end{align*}

  Max: \frac{1}{2} \text{ cake}, \frac{1}{2} \text{ doughnut}

(and many others – clearinghouse chooses!)
CEEI PROPERTIES

• **Envy-free**
  - Yes! Given the prices, you bought the best bundle you could afford
  - If you envy somebody else’s bundle, you could’ve purchased it!

• **Pareto-efficient**
  - Yes! Market is cleared → taking a Pareto step involves taking a resource from one agent and giving it to somebody new … but this lowers their utility by above

• **Strategy proof**
  - No! Intuition: CEEI clears the market → can game the system by requesting more underutilized resources
**DRF VS CEEI**

**A1: <1 CPU, 4 GB>  A2: <3 CPU, 1 GB>**

- DRF more fair, CEEI better utilization

**A1: <1 CPU, 4 GB>  A2: <3 CPU, 2 GB>**

- A2 increased her share of both CPU and memory
Two agents
Capacity: 2
Both agents will share the same preference profile:

Market clearing prices

- Don’t exist! For any price, for any item, either both agents demand that item or both do not.
- Small changes in price can cause big changes in demand
Can we tiebreak somehow?

Idea: give agents slightly different, but roughly equal budgets

- For each agent, draw budget from \([1, 1 + B]\)
- \(0 < B < \min(\frac{1}{m}, \frac{1}{(k-1)})\) – \(k\) is capacity of agent
- Note: if \(B = 0\), this is just CEEI

Still “feels fair” – random winners and losers in the budget draw, and the playing ground is still roughly equal.
A-CEEI FOR INDIVISIBLE ITEMS

Two agents
Capacity: 2
Agent 1’s budget: $1.2
Agent 2’s budget: $1

Agent 1:
???

Agent 2:
??????
A-CEEI: PROPERTIES

Always exists if $B > 0$ (need unequal budgets)

The market approximately clears:

- There exist prices that clear the market to within an error of at most $\sqrt{k*m/2}$
- Error does not depend on the number of participants → error goes to zero as a fraction of the underlying endowment

Approximately strategy proof

- “Strategy-proof in the large"

Bounded envy free

Very difficult to compute!
NEXT UP:
COURSE MATCH

PRESENTER: KOSTAS XIROGIANNOPoulos
NEXT CLASS:
INCENTIVE AUCTIONS & SPECTRUM REPACKING