CMSC 652, Complexity Theory
Fall 2017, University of Maryland

Due: September 12th, 2017
Date: September 9, 2017

Assignment 1

You must submit it electronically to ELMS. This is a group assignment. Every group only needs to submit one solution. Group members get the same credit for the group submission.

This assignment is 7% in your total points. For the simplicity of the grading, the total points for the assignment is 70.

Problem 1 [10 pts]. Let $T(\cdot)$ be an increasing computable function. Show that there exists a decidable language A such that the following occurs:

If M is a TM that, on all inputs of length n, halts in $\leq T(n)$ time, then there is an INFINITE NUMBER of x such that $A(x) \neq M(x)$. (Here we let $A(x) = 1$ if $x \in A$ otherwise $A(x) = 0$.)

Problem 2 [10 pts]. Assume $L_1, L_2 \in$ NP and $S_1 \in$ CoNP. Assume NP$\neq$CoNP. Answer each of the following with proof or a counter-example.

(a) Is $L_1 \cup L_2$ necessarily in NP?

(b) Is $L_1 \cup S_1$ necessarily in CoNP?

Problem 3 [15 pts]. Assume that the decision problem of Graph Isomorphism is in P. Show that the following function can be computed in poly-time. Given two graphs G_1, G_2,

- If G_1 and G_2 are not isomorphic than output NO.
- If G_1 and G_2 are isomorphic than output an isomorphism.

(Hint: please be careful! an intuitive approach might not work!)

Problem 4 [15 pts]. Prove that the following language

$$L = \{(M, x, 1^t) : \exists w \in \{0,1\}^t, s.t., M(x, w) \text{ halts within } t \text{ steps with output } 1\}$$

(where M is a deterministic Turing machine) is NP-complete.

Problem 5 [20 pts]. Prove that the following language is NP-complete

$$\text{3COL} = \{G : \text{there exists a coloring of } G \text{ with 3 colors s.t. no adjacent vertices share the same color.}\}$$

(Hint: You can assume 3SAT is NP-complete. You may use other resources but let me know what they are and you should hand in your own solution.)
