Lecture on October 5th, 2017
Date: October 7, 2017

Summary of Lecture 12

Reading: [Arora-Barak (AB)] Chap 6.1, 6.2, 6.5.

- Basic mathematical formulation of circuits: input, output, gates ($\mathcal{B}_0$, $\mathcal{B}_1$), size, depth.
- Languages that are recognized by circuits. The definition of P/poly.
- P/poly contains any unary language, which in turn contains undecidable languages.
- Depth-2 circuits (using $\mathcal{B}_1$) that compute any function $f : \{0,1\}^n \to \{0,1\}$ with size $O(2^n)$.
- With gates from $\mathcal{B}_0$, the above construction gives a circuit of size $O(n2^n)$ and depth $n + \log(n)$.
- For any $0 < \epsilon < 1$ and $q(n) = (1-\epsilon)2^n/n$, there exists a function $f : \{0,1\}^n \to \{0,1\}$ that cannot be computed by a circuit of size $q(n)$. The proof is by a counting argument.
- Optional parts: (1) define P by logspace-uniform families of circuits; (2) TMs that take advice;