ASSIGNMENT 1 CMSC 858K (Fall 2017)

Due by 12:30 pm on Thursday, September 14. Submit PDF via ELMS (https://myelms.umd.edu).

1. *Universality of reversible logic gates.*

   The CCCNOT (triple-controlled NOT) gate is a four-bit reversible gate that flips its fourth bit if and only if the first three bits are all in the state 1.

   (a) [2 points] Show how to compute CCCNOT using AND, OR, NOT and FANOUT gates.

   (b) [3 points] Show how to implement a CCCNOT gate using Toffoli gates. You may use additional workspace as needed. You may assume that bits in the workspace start with a particular value, either 0 or 1, provided you return them to that value. For a bonus point, give a circuit that works regardless of the values of any bits of workspace.

   (c) [4 points] Show that a Toffoli gate cannot be implemented using any number of CNOT gates, with any amount of workspace. Hence the CNOT gate alone is not universal. (Hint: It may be helpful to think of the gates as performing arithmetic operations on integers mod 2.)

2. *Product and entangled states.* Determine which of the following states are entangled. If the state is not entangled, show how to write it as a tensor product; if it is entangled, prove this.

   (a) [2 points] $\frac{2}{3}|00\rangle + \frac{1}{3}|01\rangle - \frac{2}{3}|11\rangle$

   (b) [2 points] $\frac{1}{2}(|00\rangle - i|01\rangle + i|10\rangle + |11\rangle)$

   (c) [2 points] $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

3. *Unitary operations and measurements.* Consider the state

$$|\psi\rangle = \frac{2}{3}|00\rangle + \frac{1}{3}|01\rangle - \frac{2}{3}|11\rangle.$$

   (a) [2 points] Let $|\phi\rangle = (I \otimes H)|\psi\rangle$, where H denotes the Hadamard gate. Write $|\phi\rangle$ in the computational basis.

   (b) [3 points] Suppose the first qubit of $|\phi\rangle$ is measured in the computational basis. What is the probability of obtaining 0, and in the event that this outcome occurs, what is the resulting state of the second qubit?

   (c) [3 points] Suppose the second qubit of $|\phi\rangle$ is measured in the computational basis. What is the probability of obtaining 0, and in the event that this outcome occurs, what is the resulting state of the first qubit?

   (d) [2 points] Suppose $|\phi\rangle$ is measured in the computational basis. What are the probabilities of the four possible outcomes? Show that they are consistent with the marginal probabilities you computed in the previous two parts.

4. *Distinguishing quantum states.* [6 points] Let θ be a fixed, known angle. Suppose someone flips a fair coin and, depending on the outcome, either gives you the state

$$|0\rangle \qquad \text{or} \qquad \cos\theta|0\rangle + \sin\theta|1\rangle$$

   (but does not tell you which). Describe a measurement (consisting of an orthonormal qubit basis) for guessing which state you were given, succeeding with as high a probability as possible. Also indicate the success probability of your procedure. (You do not need to prove that your procedure is optimal, but your grade will depend on how close your procedure is to optimal.)

5. *Teleporting through a Hadamard gate.*

   (a) [1 point] Write the state $(I \otimes H)|\beta_{00}\rangle$ in the computational basis.

   (b) [3 points] Suppose Alice has a qubit in the state $|\psi\rangle$ and also, Alice and Bob share a copy of the state $(I \otimes H)|\beta_{00}\rangle$. If Alice measures her two qubits in the Bell basis, what are the probabilities of the four possible outcomes, and in each case, what is the post-measurement state for Bob?

   (c) [2 points] Suppose Alice sends her measurement result to Bob. In each possible case, what operation should Bob perform in order to have the state $H|\psi\rangle$?