Standard deviations and probability of the population mean

68-95-99.7 Rule

- 68.27%
- 95.45%
- 99.73%
Bayesian Inferencing

This whole book is full of priors that have to be accepted on faith. It's a religion!

And in the public schools no less. Call a lawyer. As a math atheist, I should be excused from this.
Thomas Bayes

\[ P(A|B) = \frac{P(B|A) P(A)}{P(B)} \]
Basic Probabilities

• The probability of something occurring is the number of ways that thing can occur divided by the total number of things that can occur.

• Say you flip a “fair” coin. What’s the probability of heads?

• Ways you can have heads = 1
• Total possible outcomes = 2
• Probability of heads = P(h) = \( \frac{1}{2} = 50\% \)
Conditional Probabilities

• Conditional probabilities: If I know something ahead of time (or before), then what is the probability of event x? Conditional probabilities are calculated just like basic probabilities. What is $P(\text{drawing an Ace from a full deck}) = 4/52$. Ok. After drawing that ace what are probabilities of getting an Ace again? $P(\text{Ace|Ace}) = 3/51$. This is read what is the probability of getting an Ace given=| we’ve pulled one already. I.e., $P(A|B) = \text{The probability of } A \text{ given } B \text{ has already occurred.}$

• Conditionals = adjust all possibilities
Joint Probabilities

• What is probability of pulling an Ace of (Hearts or Diamonds)?
  • \( P(\text{Ace}) = P(a) = \frac{4}{52} \)
  • \( P(\text{Hearts or Diamonds}) = P(\text{hd}) = \frac{13 \times 2}{52} = \frac{26}{52} = \frac{1}{2} \)
  • \( P(a \text{ and } \text{hd}) = \frac{4}{52} \times \frac{1}{2} = \frac{4}{104} = \frac{1}{26} \)
  • We calculate this in our heads but are actually doing some quick multiplication.
  • Joint = multiply
Marginal Probabilities

• What is P of pulling any heart or diamond?
• \( P(\text{heart}) = \frac{13}{52} \)
• \( P(\text{diamond}) = \frac{13}{52} \)
• \( P(\text{any red}) = P(\text{heart}) + P(\text{diamond}) \)
• Marginal = add
Set A represents one set of events and Set B represents another. We wish to calculate the probability of A given B has already happened. Let's represent the happening of event B by shading it with red.

\[
\text{Blue Area} \quad \frac{\text{Red Area}}{+ \text{Blue Area}}
\]

Now since B has happened, the part which now matters for A is the part shaded in blue which is interestingly \( A \cap B \). So, the probability of A given B turns out to be:

\[
P(B|A) = \frac{P(A \cap B)}{P(A)}
\]

Therefore, we can write the formula for event B given A has already occurred by:

\[
P(B|A) = \frac{P(A \cap B)}{P(A)}
\]

or

\[
P(A|B) = \frac{P(A \cap B)}{P(B)}
\]

Now, the second equation can be rewritten as:

\[
P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}
\]

This is known as Conditional Probability.
Bayes Theorem

\[ P(A|B) = \frac{P(B|A)P(A)}{P(B)} \]
Standard deviations and probability of the population mean

68-95-99.7 Rule

- 68.27% within $\mu - 3\sigma$ and $\mu + 3\sigma$
- 95.45% within $\mu - 2\sigma$ and $\mu + 2\sigma$
- 99.73% within $\mu - \sigma$ and $\mu + \sigma$
## Type I & Type II errors

<table>
<thead>
<tr>
<th>Null Hypothesis True</th>
<th>Did we reject</th>
<th>Example</th>
<th>Outcome</th>
<th>Probability we're right</th>
<th>Probability wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t</td>
<td>They say 10 minutes is the best, we say 4 but in fact they're correct (maybe we needed more N)</td>
<td>Type 1 error</td>
<td>99.970%</td>
<td>0.030%</td>
</tr>
<tr>
<td>t</td>
<td>f</td>
<td>They say 10 minutes is the best, we say 9 but 9 is within 95% so the null hypothesis holds.</td>
<td>Great, should have accepted it</td>
<td>94.900%</td>
<td>5.100%</td>
</tr>
<tr>
<td>f</td>
<td>f</td>
<td>They say 10 minutes is the best, we say 9 but 9 is within 95% so the null hypothesis holds. We are both wrong proven with higher N or another study is conducted and finds we sampled poorly</td>
<td>Type 2 error</td>
<td>94.900%</td>
<td>5.100%</td>
</tr>
<tr>
<td>f</td>
<td>t</td>
<td>They say 10 minutes is the best, we say 4 and we're correct (we increased N later on and verified)</td>
<td>Great, should have rejected it</td>
<td>99.970%</td>
<td>0.030%</td>
</tr>
</tbody>
</table>
Some examples

• The average time it takes to find Waldo at Comic Con by a random group of people over age 6 is 7 minutes. We surveyed 30 random people over age six and they found Waldo in an average time of 5 minutes. Who is right?

• Null hypo = mean = 7

• New hypo = mean = 5. N = 30, calculated sdev = 2.08, Z-Score = -.96

• Z-table look up gives = .33*2 → p = 66%. So there’s a 66% chance that the population mean is in this range. Margin of error = 5+-0.36 66% confidence. Should we reject?
Time permitting in class examples with Waldo