1. Problem 23.4 from the textbook.

2. (Programming Assignment)

(a) Let \( z = [z_1, z_2]^T \in \mathbb{R}^2 \). We have a collection of samples \( z^{(1)}, \ldots, z^{(m)} \) where each \( z^{(i)} \) follows a Gaussian distribution \( \mathcal{N}(0, I_2) \). Moreover, we have the matrix \( A \in \mathbb{R}^{10 \times 2} \) where \( A_{ij} \) follows a uniform \([0, 1]\) distribution. Each data point \( z^{(i)} \) generates \( x^{(i)} \) according to \( x^{(i)} = Az^{(i)} + \epsilon w^{(i)} \) where \( \epsilon = 0.01 \) and \( w^{(i)} \) follows a Gaussian distribution \( \mathcal{N}(0, I_{10}) \). Construct a data set consisting of 1,000 points for \( x^{(i)} \) according to outlined generative process.

(b) Implement PCA over the generated dataset. Use the top two PCs to compute low-dimensional representations of points, i.e. for \( x^{(i)} \) find \( \hat{z}^{(i)} = [\hat{z}_1^{(i)}, \hat{z}_2^{(i)}]^T \). Make two scatter plots (one for each component) comparing the original values \( z^{(1)}, \ldots, z^{(m)} \) and the estimated \( \hat{z}^{(1)}, \ldots, \hat{z}^{(m)} \). What is the mean-squared error between the two sets? (i.e., \( 1/m \sum_i \|z^{(i)} - \hat{z}^{(i)}\|^2 \)).

Note: You can use a function from a python toolbox to do the eigendecomposition or SVD.

(c) Repeat (a)-(b) this for the case where \( x^{(i)} = Az^{(i)}z^{(i)T}b + \epsilon w^{(i)} \) where \( b \in \mathbb{R}^{2 \times 1} \) and both elements in \( b \) are generated from a uniform \([0, 1]\).